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Misconceptions in School Algebra

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Abstract
This conceptual paper aims to (1) highlight on the harmful arising from misconceptions on students' performance and achievement in algebra, (2) classifying these conceptual errors, and (3) highlighting on some past studies in this field. The authors explained the importance of algebra, and its association closely with other mathematics branches and other related subjects such as physics and economics. The authors presented the importance of revealing conceptual errors in algebra. Based on literature, the authors classified algebraic misconceptions into four categorized: algebraic expressions, linear equations, polynomials, exponents and radical expressions, and finally functions and graphs. Based on literature, a set of common conceptual errors in algebra were presented. It was emphasized that teachers should be aware of these errors and research should be expand in this field to find effective instructional strategies to address these felled algebraic misconceptions**.**
Keyword: Algebra, Algebra Misconceptions.

Introduction

Sound knowledge of mathematical concepts is the cornerstone for understanding relations, functions, and theories in various branches of mathematics. Students acquire their concepts, including mathematical ones, from the surrounding environment, classrooms, teachers, and peers. In some cases, new concepts are built inaccurately in their cognitive structures, causing a set of misconceptions to accumulate in their minds.

Holmes, Miedema, Nieuwkoop, and Haugen (2013) defined a mathematical misconception as a part of a learner’s structure that is not mathematically accurate which drive him or her presenting incorrect answers. Ojose (2015) states that misconceptions are misinterpretations and misunderstandings built on inaccurate means. It is common knowledge that these harmful misunderstandings inhibit students’ abilities and hinder their understanding of new concepts. Karadeniz, Kaya and Bozkus (2017) argued that learners stick to their misconceptions depend on them in interpreting many skills. In their empirical studies, Akhtar and Steinle (2013), Cansız, Kucuk, and Isleyen (2011), Mulungye, O'Conner, and Dr. Ndethiu (2016), and Ocal (2017) found that misconceptions have direct negative effect on students' performance and achievements.

Algebra is one of the main branches of mathematics and has many applications in the real life. Moreover, algebra is strongly related to the other mathematic branches like probability, geometry and calculus. Students in school algebra start to transit from arithmetic to abstract and focus on relations, symbols, equations, functions, representations and graphs. Mastering algebra concepts helps students to understand other branches of mathematics and other subjects that are primarily related to algebra calculations. Students with algebraic misinterpretations may face difficulties when they try to resolve problems using algebra in other branches of mathematics or other related subjects such as physics, chemistry and even economics.

Problem Statement & Study Rationale

Failure to detect and address algebraic misconceptions at some level perpetuates these conceptual errors in the cognitive structure of learners as they transit to the next level. This means that new algebraic misinterpretations will be accumulated and added to old ones which may hinder learners' understanding of mathematics. Generally, algebraic conceptual errors may be one of the main reasons of students' weakness in mathematics.

As a mathematics teacher in a secondary school, the researcher has noticed that students come from pre-secondary schools with a mix of correct and incorrect algebraic concepts, facing challenges when they learn new algebraic concepts or apply their own existing concepts in new situations. For example, students expanded $(m+n)^2$ as m^2+n^2, they distribute $2(x-5)\ as\ 2x-5$, they simplified $3y+2\ as\ 5y$, and $3a+2y\ as\ 5a^2$.

Detecting of conceptual errors in algebra is a key factor in addressing these errors. Some fruitful efforts were found in field of detecting and treating algebraic misconceptions, but more research is required in this area. (According to the researcher's knowledge), there is no research on conceptual errors for students in algebra in the United Arab Emirates where the researcher works as a mathematics teacher. Also, many past studies in literature focused on a specific set of misconceptions in algebra. In this paper, the authors seek to collect common conceptual errors reached by some previous studies in different classifications, which is an opportunity for mathematicians' researchers and instructors to view these conceptual errors and their classifications, categorizing sources of conceptual errors in algebra and thinking strategies related to these previous concepts.

Algebra Misconceptions

In literature, algebraic misconceptions were classified into four categories. These categories are algebraic expressions, linear equations, polynomials, exponents and radical expressions, and finally functions and graphs. In the following subsections, some literature will be reviewed for these four categories separately.

(1) Algebraic Expressions

In school algebra specially in presecondary and secondary schools, students start using symbols as variables and algebraic expressions to represent real life situations. In this stage, students face some difficulties regarding the meaning of a variable and how different variables may have different values. When students construct these concepts incorrectly in their minds, they cumulate different types of misconceptions in algebra starting of algebraic expressions.

Campbell (2009) observed a misconception in simplifying rational expressions, for example: students simplified $\frac{a+b}{a} = b$. Mulungye (2016) found that 37% of students simplify $\frac{a+x}{b+x} = \frac{a}{b}$. He stated that these students need to understand the meaning of algebraic expressions correctly because they committed inappropriate cancellation. He observed another common misconception when the learners simplified $\frac{1}{3x} + \frac{2}{x}$ as $\frac{3}{4x}$. Students treated the sum of denominators as a common denominator. Moreover, they confused whether $2x + 5$ is a process or an object, which was found by Irawati and Ali (2018). When students simplified $3x + 4$ as $7x$ and $4 + 3x^2$ as $7x^2$, they described this conceptual error as considering the (+) symbol as invitations to do something (Chow, 2011). Mulungye, O'Connor, and Ndethiu (2016) argued that students supposed that the answer should not contain a sign (operator symbol); they usually finished them by simplification. The study found that teachers' instructional strategies did not treat students' conceptual errors and relevant their deficiencies in teaching algebra. According to Mulungye, O'Connor, and Ndethiu (2016), teachers need assistance in misconceptions identifications and how these misunderstanding could be built in the whole learning process. Irawati and Ali (2018) described this common misconception as merging the algebraic addition (conjoining) incorrectly.

Homles, Miedema, Nieuwkoop, and Haugen (2013) distinguished between conceptual errors in algebraic expressions and computation errors. They described stating $3y^2.2 = 6y$ or $4.5a = 21a$ instead of $6y^2$ and $20a$ respectively as computation errors and stating $3a.2 = 5a$ or $4.5b = 9b$ as conceptual errors. They stated that in case of conceptual errors, teachers need to detect the misconception involved and treat them while communicating with students, concerning computation errors require teachers to alert students to their mistake. Regardless of this classification, the authorss of the current study, as a mathematics teacher, and from literature, argue that these misconceptions are not common. For example, a common misconception was noticed when students simplify $4x + 3$ as $7x$ where stating $3x.2 = 5x$ not common (Irawati & Ali, 2018; Mulungye, O 'Connor, & Ndethiu, 2016).

Campbell (2009) observed that some students confuse operations, for example: they worked $3 + x^2$ as $3ax^2$. Students misinterpret the meaning of variables and thus join algebraic 'objects' as a new one 'object' e.g. $2x + 5y = 7xy$. According to Luka (2013), students had a misconception of over simplification when they were given the question: subtract $3x$ from 5. They wrote "2 or $2x$" as a correct answer, while others answered with reversal error and wrote $3x - 5$.

Dodzo (2016) observed that some students merged algebraic addition incorrectly, which was noticed by (Booth, Barbieri, Eyer, and Blagoev, 2014; Irawati and Ali, 2018; Mulungye, O 'Connor, & Ndethiu, 2016). He found that students simplify $2x + 5$ as 7. They ignored variables instead of operating like terms. An interviewee thought that the letter "x" can be considered or not. She claimed that either way has the same meaning. Another student thought that he can collect the like terms 2; 4; 7 and 2 to simplify $2m + 4n + 7 + 2m$ and then added them to get 15 as a simplest form. Dodzo (2016) exposed other misconceptions for students in algebra, there were as follows:

(a) Wrong simplification: $m \times \frac{n}{m} = \frac{mn}{m}$ as a final answer. An interviewee said "m in numerator is squared, so they are not like terms".
(b) Incorrect denominator: $\frac{mn}{xy} + \frac{1}{y} = \frac{mn+1}{xy^2}$. Students multiply x and y^2 to get common multiple and add the numerators mn and 1. The same misconception was observed by Irawati and Ali (2018). Most students find $\frac{m}{2} + \frac{m}{3}$ as $\frac{m^2}{5}$ by multiplying numerators and adding denominators.
(c) Over simplification: some students wrote $\frac{xa+xb}{xb+xc}$ as $\frac{x(a+b)}{x(b+c)}$ and then $\frac{a}{c}$. An interviewee said "like terms mast be canceled on the numerator and denominator. Luka (2013) found the same misconception related to using factorisation to simplify algebraic expression like $\frac{ax+bx}{x+cx} = \frac{abx^2}{cx^2}$. Students multiply the terms on numerator and dominator separately instead of factorising them. Also, students ignored the order of operations rules (Chow, 2011). Some students wrote $3 + x \times 2$ as $6x$. They worked the problem from left to right.

(2) Linear Equations

Students solve linear equations using their previous knowledge of. In case of algebraic expressions misconceptions, students will face difficulties in solving linear equations. Students also may have some misinterpreting about the procedures that are usually used to solve this type of equations like inverse operation. In this section, the authors will present some past studies related to common misconceptions that students have in linear equations.

Toka and Askar (2002) found a misconception related to using distributive property incorrectly. Some students rewrote the equation $5 - 3\,(2 - x) = -7$ as $5 - 6 - 3x = -7$. Others wrote the same equation as $2(2 - x) = -7$, using the order of operations inaccurately. Also, a conceptual error related to distributing minus signs was found, for example: 2 – (3x – 4y) = 2 – 3x – 4y. Steinle, Gvozdenko, Price, Stacey, and Pierce (2009) stated that some students treat the letter x in the equation $x + x + x = 15$ as they do with empty boxes ($\square + \square + \square = 15$), choosing 3, 6, 6 or 9, 3, 3 as the values of x, x, x respectively. Bardini, Vincent, Pierce, and King (2004) justified this misconception as a misunderstanding of the rule of x in means different number. They found that some students omitted the choice "6,6" when they solved the equation: $x + y = 12$, and chose "4, 8" or "7, 5" as the correct answer. Chow (2011) stated a type of misconceptions related to missing literal symbols as variables; many responded "never" when they asked to determine when $a + b + c = a + z + c$ is correct. An interviewee explained "different letters mean different values", which was observed by (Bardini, Vincent, Pierce & King, 2004) and (Li, 2006). Li (2006) described this conceptual error as a sound understanding of "variable" as "place holder". Students may think that different letters should represent different numbers.

Li (2006) observed that when some students treat the equation like $10 = 3 + 5x$ incorrectly, they misunderstood the structure of $3 + 5x$ as $3 + 5 + x$, not being aware that "$3 + 5x$" was the same as "$3 + 5 \times x$". They might think of the omitted sign "$\times$" as "+". Booth and Koedinger (2008) categorized the linear equations misconceptions into two categories: procedural misconceptions such as combining non-like terms, using the inverse operation incorrectly and committing a negative, misconceptions such as equal signs and negative signs.

Chow (2011) observed that some students removed a term from both sides of the equation by subtracting it nevertheless of the adjoining operator symbol (+ or -). They worked $x - 5 = 4$ as $x - 5 - 5 = 4 - 5$ and then, $x = -1$. Some students also used inverse operation incorrectly; they solved $5 = 7x$ by selecting the option $7 \div 5$ instead of $5 \div 7$. Students realized the need to isolate the variable, but were choose needed inverse operation inaccurately. Dodzo (2016) found that students rewrote the equation $1 - 2x = 13$ as $1 - 13 = 2x$ and then, $12 = 2x$, he named this misconception "inverse error". An interviewee said "the difference between 1 and 13 is 12. He also observed what he named "omission error" in which some students rewrote the equation $2x - 5 = 10 - 3x$ as $2x - 5 - 5 = 10 - 3x$, and then $2x = 10 - 3x$. An interviewee said "this is because $-5 - 5 = 0$. The authors of the current paper suggest that another question should be asked to the interviewees related to adding or subtracting a number from only one side of the equation. Some students move an item from one side to another without changing the sign (Booth, Barbieri, Eyer, & Blagoev, 2014). Fore example, moved $5y$ in the equation $2 + 5y = 1 - 7y$ to the right side without changing (+) sign to (-) sign. They also observed that students had a negative sign error when they solved a linear equation like $7 = 3 - 2y$. They subtracted 3 from the both sides and ignored the (-) sign to get $4 = 2y$. One more misconception was found when students chose the operation incorrectly when they solve an equation like: $3\,(\,5x + 2) = 8$ as they transported "3" to the right side of the equation: $(5x + 2) = 8 - 3$, using subtraction instead of division. According to the authors of the current study, the researcher of previous study named this misconception inaccurately "transporting error" instead of "inverse operation error". Students also used the wrong operator when they tried to solve an equation like $\frac{5-3y}{4} = 5$, subtracting 4 from both sides, instead of multiplying by 4 (Booth, Barbieri, Eyer, & Blagoev, 2014).

Mulungye (2016) detected that students used the positive sign, the negative sign and the equal sign incorrectly when they solved the equation $13 - 6x = 4 - 2x$, their response was $8x = 17$ or $9 = -4x$.

(3) Polynomials, Exponents and Radical Expressions

Mulungye, O 'Connor, and Ndethiu (2016) are interested in the most commonly misconceptions happening in high schools' classrooms. They found that students expand $(x + y)^2$ as $x^2 + y^2$. Campbell (2009) described this misconception as over-generalising, including false-linearity. Booth, Barbieri, Eyer, and Blagoev (2014) observed that students start correctly when they expanded $(y + 4)^2$.They worked the problem as $(y + 4)(y + 4)$. The misconception appeared in the second step where they wrote y^2+ 16 as a final answer. They stated that students did not distribute entire binomial to entire binomial. Luka (2013) reveled the same misconception and described it as a misinterpretation of distributive law in which $a(b + c) = ab + ac$. Students intuitively misuse the rule in similar situations because the formal distributive property of multiplication over addition was deeply precipitated in their mind. For example, students simplified $3(a - b)$; their response is $3a - b$. Students are obligated either to 'close' their answer or to overlook the parentheses and work from left to right. Also, students simplified $4\ (\ x+3)$ as $4x+3$. When students were asked to simplify $(6x-2)(4x\ +3)$as $24x^2\ +\ 18x-8x\ +\ 6$ instead of $24x^2+18x-8x-\ 6$.

Bush (2011) observed other misconceptions when students try to simplify the expression $3(5x\ -\ 2)\ +\ 2(4x\ -\ 1)$. One of them was the using the negative sign incorrectly. Also, some students made commotional errors with positive whole numbers. There were some students who tried to perform inverse operation though it was not an equation; others made a transcription error within the steps of the problem. Incorrect use of signs, combining like terms incorrectly, omitting a negative, and difficulty with distributive property were observed in this item. Ojose (2015) found that some students add powers in case of adding exponents. He stated that students think incorrectly that they can add the powers because both terms have the same base; they simplified $y^4\ +\ y^4$ as y^8. Campbell (2009) observed that some students operated on one part of a compound term, for example: $(2y)^2\ =\ 2y^2$.

A'yun and Lukito (2018) found a misconception related to second degree radical addition. Students worked $\sqrt{x^2+y^2}$ as $x+y$ which was found by Mulungye (2016).

(4) Functions and Graphs

Students from elementary education to university in some way come across the concept of function and perform activities about this concept (Casnsiz, Kucuk, & Isleyen, 2011). Students perform activities about functions use their knowledge about algebraic expressions, exponents, polynomials, radicals and equations. Different types of functions can be visualized using graphs to interpret their behaviour under different conditions (Ocal, 2017). The following are some common misconceptions about linear functions and their graphs.

Cansız, Kucuk, and İsleyen (2011) observed some misconceptions about functions and their graphs. They observed that some students can not determine whether a given graph is a function, they misunderstand the definition of the function. Some students combined the lines that was given in the graph and then decided that graph was a function. The researchers argued that these students had a misconception about "continuity". Also, it was observed that some students thought that the graph cannot be a function if it is not continuous. They also observed that some students cannot distinguish between the independent variable and dependent variable for a given function or it's graph.

Li (2006) observed a conceptual error named "a misconception at the stage of process-oriented thinking" in which students consider only one variable of a function. They tended to ignore the independent variable. For example, they only considered the differences between values of the dependent variable and didn't consider the value of independent variable in order to determine whether given values represent linear function. The authors of the current paper argued that this was not misconception. The values of dependent variable are enough to determine whether the given values represent linear function; that is, if the first differences of the values of dependent variable are equal, then the variables form a linear function.

Ocal (2017) found a misconception related to asymptotes for $\frac{1}{x}$, $\ln x$ and e^x functions graphs. Students had roughly sketched them and did not give explanations about their sketches. Bush (2011) found some misconceptions when students were asked to interpret the graph shown below, estimate the water level rises in feet, between 1 minute and 4 munities and then use the reasoning to expect the water level outside the area shown in the graph:

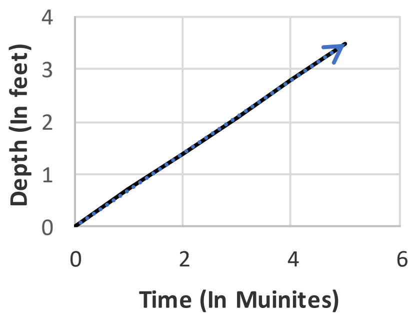

Figure1: Depth of water in poo

Students used incorrect symbolism in their explanation to count with patterns in the graph and generalizing. The researcher found a misconception related to the interpreting and predicting of the graph such as estimating the feet. Students were asked to do the following item: “Khaled sold 12 tickets to a school play. Khaled's total sales ‘t’ for the tickets is given by the formula: $12 \times c = t$, where c is cost per ticket. What were Khaled’s total sales if the cost of each ticket is $51?”. Students substituted the wrong value in the equation and used addition or division instead of multiplication.

Conclusion

As it was shown, misconceptions in algebra are common. Several empirical studies showed many types of conceptual errors in four categorizes: (a) algebraic expressions, (b) linear equations, (c) polynomials, exponents and radical expressions, and (d) functions and graphs. Students as interviewees showed sticking of their existing concepts and they provided incorrect explanations for each misconception they have. The negative effects of misconceptions in algebra were emphasized on student performance and achievement in algebra and on other related subjects which makes the detection of these errors an urgent necessity in the way they are addressed.

The authors recommended that conceptual change should be part of the learning process in case of misconceptions. Mathematicians specially teachers and instructors need to aware of students’ misunderstandings, detecting these misinterpretations by providing critical situations. Research in this field should be expanding to find effective instructional strategies to treat different types of misconceptions in algebra and other branches of mathematics.

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