Mathematical Analysis of Retirement Income Benefit Based on Annuitization Approach

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Abstract
etirement income inadequacy is one of the major problems to the private sector workforce with defined contribution retirement scheme. It is believed the existing system of Employees Provident Fund (EPF) in Malaysia adopted of lump-sum payment of their retirement benefits should be replaced by annuity-based scheme which can control the total retirement funds balance of the members. Using a mathematical analysis, the comparing system of retirement benefits by EPF can be analyzed. It is clearly showed that, with annuity-based scheme provide the retirees with a steady income stream for the next 20 years after retirement age.
Keywords: Mathematical Analysis, Finite Series, Retirement System, Annuity

1.0 Introduction
Retirement system basically is classified as three tiers system. Many countries nowadays have adopted these three tier pension or retirement income systems in which comprises of social security, employment-based pensions, and individual savings (Moore, 2011). In fact, the major system of retirement adopted in Malaysia can be only classified as two tier pension system, which consists of Employee Provident Funds (EPF) and pension's scheme for the government servant. Both of the schemes can be known as employment-based system. As a developing country, Malaysia is still left behind by other country in the same class. As we can see Chile, Brazil, China, and India are listed in a top 20 countries have a good retirement system in the world (Mercer, 2015). Malaysian people still lack of awareness to think about their retirement saving preparedness (Ibrahim, Isa, & Ali, 2012). Furthermore, the low-income and middle-income earners in Malaysia are facing serious problem in the adequacy of retirement resources and savings (Folk, Beh, & Baranovich, 2012). According to Poh (2015),

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around 70% of the EPF members at the age of 54 have less than RM50,000 in their EPF account, and about 50% of the retirees deplete their EPF savings for less than 5 years.

In light of these issues, this study aims to focus on the form of benefit payment that can be done by EPF to the prospect retirees. As the lump-sum payment is proven cannot guaranteed to sustain the retiree's financial position during retirement age. Then it is important to construct a new retirement benefit of annuity-based system for private sector especially for those in low-income and middle income categories. Annuity schemes basically cater for those in high-income and upper-middle income earners, but it is not possible to offer for low-income and middle-income as well since the accumulated value which initiated from small amount of contribution can reaches a big amount of money when it comes maturity (Ghazali et al., 2015). At least this scheme can guarantees the financial position among elderly by providing a sustainable monthly income stream like what received by the government servant.

2.0 Methodology

First and foremost, the general mathematical formulation of accumulated retirement savings from EPF account's member will be constructed. Then, it will apply into targeted income group member which basically having a problem with financial aspect after retirement. Thus low-income and middle-income earners will illustrate the constructed formula.

Furthermore, by using a mathematical analysis of finite series, a benchmark of ideal accumulated retirement savings will be constructed to measure whether such accumulation retirement wealth from first part would be appropriate to apply annuity-based system to replace the traditional retirement benefit payment to the EPF members.

3.0 Result And Discussion

The sample of EPF profile's member will be used in order to derive the mathematical description of total accumulated retirement savings from EPF, table 3.1.

Table 3.1:
Profile of EPF Member

1	Monthly salary	RM x
2	Working period	25 year – 55 year (30 years)
3	Contribution rate,	8% by employee 13% by employer 21% of total contribution rate per month
4	Monthly contribution amount, C	Contribution rate x monthly salary
5	Expected EPF rate of return, r	5.0%

The general formula of accumulated fund can be constructed by calculating the accumulated fund for the first three year:

Accumulated Fund Year 1, AF_1 = C

Accumulated Fund Year 2, AF_2 = C + AF_1(1+r)
$$= C + C(1+r)$$

Accumulated Fund Year 3, AF_3 = C + AF_2(1+r)
$$= C + [(C + AF_1(1+r)](1+r)$$
$$= C + [(C + C(1+r)](1+r)$$
$$= C + C(1+r) + C(1+r)^2$$

$$= C[1 + (1+r) + (1+r)^2]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{equation (1)}$$

$$0 = C[1 + (1+r) + (1+r)^2] - AF_3 \ldots\ldots\ldots\ldots\ldots\text{equation (2)}$$

Multiply both side of equation (1) with $(1+i)$,

$$AF_3 (1+r) = C[1 + (1+r) + (1+r)^2] (1+r)$$

$$0 = C[(1+r) + (1+r)^2 + (1+r)^3] - AF_3 - AF_3(1+r)\ldots.\text{equation (3)}$$

Subtract equation (3) with (2):

$$C(1+r)^3 - AF_3 r - C = 0$$

$$AF^3 = C\frac{(1+r)^3 - 1}{r}$$

Therefore the general formula of accumulated fund is actually formula of conventional future value:

$$AF^n = C\frac{(1+r)^n - 1}{r}$$

3.1. Illustration of Low-income Earners

In Malaysia, the minimum income should be at least RM1,000. Therefore, EPF member's profile for low-income earners can be illustrated as follows;

1	Monthly salary	RM1,000
2	Working period	25 year – 55 year (30 years)
3	Contribution rate	8% by employee 13% by employer 21% of total contribution rate per month
4	Expected EPF rate of return, r	5.0%

Thus, the accumulated fund for 30 years saving period can be determined by using the above formula:

Total Contribution, C = Contribution rate x Annual salary

$$= 21\% \text{ x (RM1000 x 12)}$$
$$= 0.21 \text{ x RM12,000}$$
$$= \text{RM2520}$$

$$AF^{30} = 2,520\frac{(1 + 5.0\%)^{30} - 1}{5.0\%}$$
$$= 167,426$$

3.2. Illustration of Middle-income Earners

In Malaysia, the minimum income should be at least RM5,000. Therefore, EPF member's profile for middle-income earners can be illustrated as follows;

1	Monthly salary	RM5,000
2	Working period	25 year – 55 year (30 years)
3	Contribution rate	8% by employee 13% by employer 21% of total contribution rate per month
4	Expected EPF rate of return, r	5.0%

Thus, the accumulated fund for 30 years saving period can be determined by using the above formula:

Total Contribution, C = Contribution rate x Annual salary

$$= 21\% \text{ x (RM5000 x 12)}$$

$$= 0.21 \times RM60,000$$
$$= RM12,600$$

$$AF^{30} = 12,600\,\frac{(1 + 5.0\%)^{30} - 1}{5.0\%}$$
$$= 837,129$$

The accumulated fund calculated using the above formula are RM167,426 and RM837,129 for low-income and middle-income groups respectively. These figures will accumulate with a conditions of having no pre-withdrawal before getting retired. In fact, most of the EPF members would choose the options to withdraw all retirement fund whenever reach 55-year-old. This however brings about depletion for the first five years and need to look for a new job again. Alternatively, the lump-sum benefit payment can be changed into monthly basis payment until the total retirement fund depleted into zero, likes annuity-based scheme.

3.3. Measurement Level for Accumulated Retirement Savings in Annuity-based Scheme

Basically, using annuity-based scheme the retirees will entitle with a series of monthly income, as their total accumulated retirement funds will be given on monthly basis instead of lump-sum figure. As for example, if the retiree has a total RM100,000 of accumulated retirement funds, he will receive a monthly income, let say RM1,000 for 100 months consecutively.

In this part, it is essential to set up a measurement level of total retirement wealth to have a sustainable annuity-based scheme. Statistically have shown that life expectancy of Malaysian in 2015 is around 75 years (Department of Statistic Malaysia, 2015). Therefore, the retirees of 55-year-old would expect to live for 20 years more. It is clearly depicts that any annuity-based schemes should offer a 20 years retirement benefit payments to their participant.

Suppose that people who retire with income RM1,000 need the same amount to keep their lifestyle, and the monthly annuity payment would be appropriate to give them RM1,000 per month from their total accumulated retirement savings. In addition, the balance of their savings will entitle the profit return of 5%. For each annual annuity payment can be discounted to the first year of retirement in order to get to appropriate figure of accumulated retirement funds for 20 years. Therefore, the suggested retirement savings, SRS, can be derived as follows,

$SRS = Year_1 \times (1 + r) + Year_2 \times (1 + r)^{-2} + Year_3 \times (1 + r)^{-3} + ... + Year_{20} \times (1 + r)^{-20}$
$\quad = 12,000\,(1 + 5\%) + 12,000\,(1 + 5\%)^{-2} + 12,000\,(1 + 5\%)^{-3} + ... + 12,000\,(1 + 5\%)^{-20}$

Thus, it can be simplified like this

$$\sum_{n=1}^{n}\frac{12,000}{(1+r)^{n}}$$

Hence the actual retirement savings, ARS should accumulate more than suggested retirement savings. From here, the SRS for low-income earners and middle-income earners can be simulated as table 3.2 and 3.3 below,

Table 3.2: Appropriate Measurement of Retirement Funds for Low-income Earners

Year	Annuity Payout	Discounted Annuity Payout	Total SRS
1	12,000	11,429	11,429
2	12,000	10,884	22,313
3	12,000	10,366	32,679
.	.	.	.
.	.	.	.
.	.	.	.
18	12,000	4,986	140,275
19	12,000	4,749	145,024
20	12,000	4,523	149,547

Table 3.3: Appropriate Measurement of Retirement Funds for Middle-income Earners

Year	Annuity Payout	Discounted Annuity Payout	Total SRS
1	60,000	57,143	57,143
2	60,000	54,422	111,565
3	60,000	51,830	163,395
.	.	.	.
.	.	.	.
.	.	.	.
18	60,000	24,931	701,375
19	60,000	23,744	725,119
20	60,000	22,613	747,733

From the table 3.2 and 3.3, we have set up a benchmark to measure appropriate retirement savings for low-income and middle-income earners. Now we can compare the figures calculated using the formula in section 3.1 and 3.2 with the value SRS in the table 3.2 and 3.3. Both the actual retirement savings calculated using the formula for low-income and middle income earners have exceeded the total accumulated funds in suggested retirement savings. These indicate the annuity-based system can provide a steady monthly income stream for at least 20 years after retirement age.

5.0 Conclusion

In this case, the annuity-based scheme can guarantee to support financial needs among elderly for at least 20 years period, following Malaysian life expectancy. Thus, it is a good suggestion to the EPF to provide an annuity-based system benefit as a compulsory to their prospect retirees instead of giving a lump-sum benefit payment in which most of retirees finish to spend it within 5 years period. Nevertheless, it is good news for EPF members to maintain their income after retirement for almost 20 years, as it is important to keep their basic needs and other expenses during retirement, but there are other important exposures of the retirement hazards like's costs of healthcare, presuming a longer working life, misjudging how long one spouse will live, demographic change and consequences to

retirement (Sharii, Yusoff, & Shahizan, 2013). Moreover, Folk, Beh, & Baranovich (2012) have mentioned that most of the elderly people aged 60 and above have spent more on their healthcare costs which is six times a year, on an average. The costs for medical expenses would be a greater concern here as it would be a rising costs for the whole society (Ghazali, et al., 2017). Therefore, it would be an interesting for future research to study as to whether the annuity payment can afford to cover the additional healthcare costs, besides their basic needs likes cost of livings.

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