

Evaluation of Creating Scoring Opportunities through Trajectory Analysis for Goal Scoring in Football

Dalia Attan¹, Razif Sazali^{2*}, Muhammad Haziq Amir Hamzah¹

¹Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA (UiTM), Negeri Sembilan Branch, Seremban Campus, 70300 Seremban, Negeri Sembilan, Malaysia, ²Faculty of Sport Science and Recreation, Universiti Teknologi MARA (UiTM), Cawangan Negeri Sembilan, Kampus Seremban, Malaysia

*Corresponding Author Email: razifsazali@uitm.edu.my

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Abstract

Determining the optimal launch angle for penalty kicks has practical value for coaches and performance analysts, yet the reviewed literature did not identify a computational study that applied projectile motion modelling to establish angle-specific thresholds under standard FIFA constraints simultaneously for both vertical and horizontal success criteria. This study analysed the trajectory of a football during penalty kicks using Newtonian mechanics and MATLAB simulation, testing eight launch angles ($\theta = 10^\circ$ to 45°) at a fixed initial velocity of 24.3 m/s. For each angle, maximum vertical displacement ($y_{\max}$), ball height at the goal line (y_{11}), and total horizontal range (x) were computed analytically using equations derived from Newton's Second Law and verified numerically in MATLAB. The goal-scoring criterion required simultaneous satisfaction of two conditions: $y_{11} < 2.44$ m (below the FIFA-standard crossbar at $x = 11$ m) and $x > 11$ m (beyond the penalty mark). Only angles of 10° and 15° satisfied both criteria. At 10° , ball height at the goal line was 0.90 m, providing a 1.54 m safety margin below the crossbar; at 15° , goal-line height was 1.87 m with a 0.57 m margin. All six remaining angles (20° to 45°) violated the vertical criterion, with goal-line height rising from 2.87 m at 20° to 8.99 m at 45° . The horizontal criterion was satisfied at all eight angles, confirming that horizontal reach is never the binding constraint at this velocity. Within the scope of the reviewed literature, a contribution of this study is the explicit analytical separation of $y_{\max}$ (peak trajectory height) from y_{11} (ball height at the goal line), which produce different practical conclusions. These findings establish a quantified angular window ($0^\circ < \theta \leq 15^\circ$) for penalty kick success, provide coaches with an evidence-based training target, and demonstrate the utility of computational physics modelling as a tool for sports performance analysis.

Keywords: Penalty Kicks, Projectile Motion, Football Trajectory, Goal-Scoring Strategies, Matlab Simulation, Computational Biomechanics

Introduction

Football is the world's most widely played sport, and the penalty kick is one of its most analytically tractable events. Unlike open-play situations involving complex team dynamics, a penalty kick occurs under precisely standardised conditions: the ball is placed 11 metres from the goal line, the goalpost stands 2.44 metres high, and the kicker operates with a fixed target. This controlled scenario renders the penalty kick uniquely amenable to biomechanical and computational analysis.

Despite its apparent simplicity, the penalty kick has a historically inconsistent success rate in high-stakes competition. Coaches and athletes frequently rely on intuition and repetition rather than evidence-based strategies grounded in the physics of ball flight. Yet the trajectory of a kicked football obeys well-established laws of projectile motion: a parabolic path determined by initial velocity, launch angle, and gravitational acceleration. Understanding these variables mathematically provides a direct pathway to optimising technique.

Previous research has demonstrated the utility of physics-based trajectory analysis across multiple sports. Nor-Al-Din et al. (2021) applied Newton's Second Law to derive optimal throwing angles for basketball free throws, demonstrating that a narrow angular window maximises scoring probability. Phomsoupha and Laffaye (2015) showed that badminton shuttlecock trajectory is sensitive to angular and velocity parameters, with small deviations producing significant accuracy losses. Blair et al. (2020) established kinematic evidence that lower kick-leg angles at contact produce more accurate and lower-trajectory shots in Australian football, a finding transferable to the penalty kick context. In football trajectory modelling, Goff (2013) provided a comprehensive review of aerodynamic effects on sport projectiles, and Ortega Contreras et al. (2018) derived modified trajectory equations for projectiles in resistive media, confirming that flight time and optimal angle are sensitive to aerodynamic conditions. Javorova and Ivanov (2018) modelled the flight of a football during direct kicks, identifying drag, lift, and gravity as the primary forces shaping trajectory.

A possible analytical gap in the reviewed literature is the treatment of the penalty kick goal-scoring criterion. Among the reviewed studies, the maximum trajectory height ($y_{\max}$) is typically reported as the determining variable, but the operationally correct criterion is the ball's height at the goal line (y_{11}) when it crosses $x = 11$ m. The two quantities differ because the ball does not reach its apex before the goal line at most realistic angles and velocities. The reviewed literature did not identify any study that made this distinction explicit or reported y_{11} as the primary criterion. This apparent gap matters practically: using $y_{\max}$ overstates the failure magnitude at moderate angles and understates the safety margin at successful angles. This study addresses this apparent gap. Applying Newtonian mechanics and numerical verification in MATLAB, it systematically models the relationship between launch angle, maximum vertical displacement, ball height at the goal line, and horizontal range across eight angles from 10° to 45° . The objectives are:

- (1) To compute ball height at the goal line (y_{11}) and maximum vertical displacement ($y_{\max}$) for each launch angle, and to demonstrate analytically why y_{11} is the correct criterion for goal-scoring evaluation.
- (2) To confirm whether horizontal range constitutes a binding constraint for penalty kick success at the tested initial velocity.

- (3) To determine the optimal angular window for penalty kick success and quantify the technical margin available within that window.

Literature Review

Projectile Motion in Sports: Foundational Evidence

Projectile motion, defined as the parabolic flight of an object under gravity, governs ball flight across a wide range of sports. The overarching principle is that for a given initial velocity, there exists a narrow angular window that maximises the probability of achieving the desired outcome, whether clearing a crossbar, passing through a hoop, or landing in bounds. Deviating from that window, even by small amounts, produces disproportionately large errors at the target.

In basketball, Nor-Al-Din et al. (2021) derived the relationship between throwing angle and free-throw accuracy using Newton's Second Law, finding that precise angular control within a relatively narrow band maximises the probability of the ball clearing the rim. Their study was notable for applying a calculus-based model to optimise both angle and initial velocity simultaneously as a function of player height. Phomsoupha and Laffaye (2015) documented the biomechanics of badminton, demonstrating that initial velocity and launch angle jointly determine shuttlecock trajectory accuracy in smash strokes at speeds commonly exceeding 200 km/h. Blair et al. (2020) identified lower kick-leg angles at contact as a predictor of goal-kicking accuracy in Australian football, establishing a direct kinematic link between technique and the ball's vertical trajectory.

Goff (2013) reviewed two decades of aerodynamics research on sport projectiles, discussing vacuum physics, air drag, lift, and computational fluid dynamics across 18 sports including football. His review confirmed that the aerodynamic forces most significant for football shots at typical velocities are gravity and drag, with spin-induced Magnus forces adding complexity that varies with the ball's rotational speed. Ortega Contreras et al. (2018) introduced conformable derivative solutions for projectile motion in both free and resistive media, demonstrating analytically that trajectory, maximum height, flight time, optimal angle, and maximum range all depend on the resistive parameter and deviate systematically from idealised predictions. Their work provides the theoretical framework within which the idealised model of the present study sits as a limiting case.

Football Trajectory Analysis: Penalty Kicks and Direct Kicks

Within football, Javorova and Ivanov (2018) studied the flight trajectory of a football for direct free kicks and corner kicks, modelling the ball as a rigid body subject to gravity, aerodynamic drag, and the Magnus effect. Their work identified the drag coefficient and lift coefficient as the critical aerodynamic parameters shaping trajectory and demonstrated that for shots aimed at specific goal zones, the interaction between initial linear and angular velocity determines whether the ball reaches the target. Although their focus was on the corner kick and direct free kick rather than the penalty kick, their treatment of the governing equations and their identification of the most dangerous goal zones (upper corners farthest from the goalkeeper) are directly informative for penalty kick analysis.

Leela and Comissioning (2011) modelled football penalty kicks mathematically, analysing the variables involved and devising shooting strategies to achieve high success rates. Their

approach used projectile motion equations and numerical methods to relate feasible ranges of angles to corresponding velocities, with the explicit aim of informing coaches about the techniques required to produce successful trajectories. They identified launch angle and initial velocity as the two fundamental parameters of a penalty shot and demonstrated that specific combinations of these parameters produce trajectories passing within the goal frame. Their work is the most direct predecessor of the present study in scope, though it did not systematically enumerate angle-specific thresholds or distinguish $y_{\max}$ from y_{11} .

Blair et al. (2020) examined the kinematic predictors of goal-kicking accuracy in Australian football using a three-dimensional inertial measurement system on 18 players performing 15 goal-kicks each. Accurate goal-kicks were characterised by a straighter approach line, less kick-leg range of motion, lower linear velocity at contact, and greater ankle plantarflexion at the follow-through. The study established that lower kick-leg angles at contact consistently produced more accurate, lower-trajectory shots, providing biomechanical grounding for the computational prediction that lower launch angles (10° to 15°) are optimal for penalty kicks in association football.

Ciacchi et al. (2022) analysed the kinematics of shot put among Italian athletes, demonstrating that release angle interacts with initial velocity to determine projectile range in ways well predicted by standard kinematic equations. Their finding that optimal angles in practice are substantially below 45° , due to the dependence of projection velocity on angle in human biomechanics, which provides transferable context for understanding why penalty kickers operating under technique-specific constraints may naturally find lower angles more tractable.

The Case for a Computational Approach

The studies reviewed above converge on two principles relevant to the present study. First, the critical variable in penalty kick success is vertical trajectory control, not horizontal power. At standard penalty velocities, the ball easily reaches the goal line regardless of angle; the binding constraint is whether it remains below the crossbar. Second, the correct measure of this constraint is the ball's height when it crosses the goal line, not the peak height of the trajectory, because the apex of the trajectory lies beyond the goal line for all practical angles at this velocity.

The reviewed literature did not identify any computational study that made this distinction explicit or systematically reported y_{11} as the primary criterion. The present study addresses this apparent gap by deriving both $y_{\max}$ and y_{11} analytically, comparing them, and demonstrating that y_{11} produces different and more practically relevant estimates of the available safety margin.

Methodology

Study Design

This study employs a computational modelling approach grounded in classical Newtonian mechanics, structured across three sequential phases: (i) Data Acquisition, (ii) Mathematical Modelling, and (iii) MATLAB Simulation and Evaluation. No human subjects were involved; all input parameters were derived from established physical constants and peer-reviewed literature. The analytical workflow is illustrated in Figure 1.

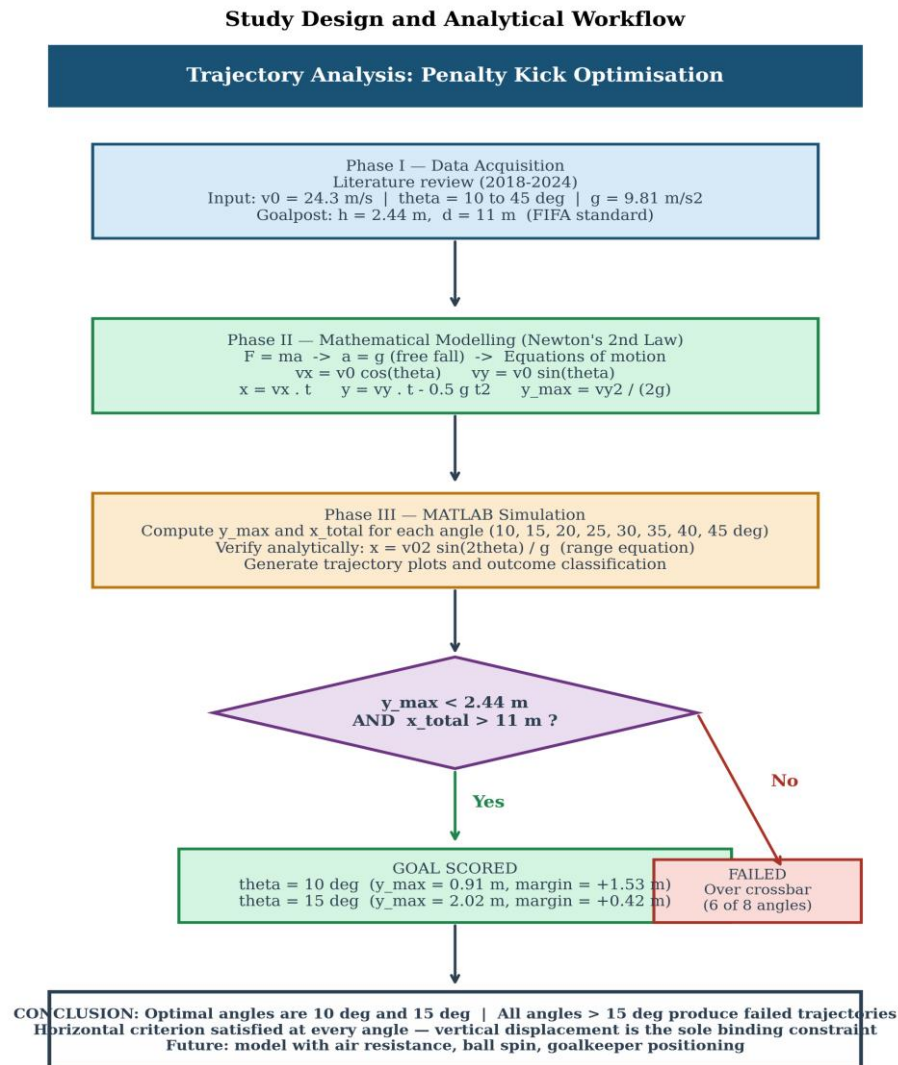


Figure 1. Study design and analytical workflow. Phase I establishes input parameters from the literature. Phase II derives the governing equations of projectile motion. Phase III computes outcomes in MATLAB and classifies each angle against the dual success criteria.

Mathematical Foundation: Equations of Projectile Motion

The framework derives from Newton's Second Law of Motion:

$$\mathbf{F} = m\mathbf{a} \quad (3.1)$$

Since acceleration is the rate of change of velocity:

$$\mathbf{a} = d\mathbf{v}/dt \quad (3.2)$$

Combining and integrating yields the kinematic velocity equation:

$$\mathbf{v}_f = \mathbf{v}_0 + \mathbf{a}t \quad (3.3)$$

For a projectile in free fall, the only net force is weight $W = mg$, where $g = 9.81 \text{ m/s}^2$. This gives $a = g$, and vertical deceleration is governed solely by gravity. At launch angle θ , the initial velocity decomposes into orthogonal components:

$$v_x = v_0 \cos(\theta) \quad (3.4)$$

$$v_y = v_0 \sin(\theta) \quad (3.5)$$

Total horizontal distance, derived from the range equation, is:

$$x = (v_0^2 \sin 2\theta) / g \quad (3.6)$$

Vertical displacement at time t :

$$y = v_y \cdot t - \frac{1}{2} g t^2 \quad (3.7)$$

Maximum vertical displacement (at $t_{\max} = v_y/g$):

$$y_{\max} = v_y^2 / (2g) \quad (3.8)$$

Ball height at the goal line (at $t_{11} = 11/v_x$):

$$y_{11} = v_y (11/v_x) - \frac{1}{2} g (11/v_x)^2 \quad (3.9)$$

Equation (3.9) is the operationally critical quantity: the ball's actual height when it crosses the goal line at $x = 11$ m. At $v_0 = 24.3$ m/s, the ball has not yet reached its apex when it crosses the goal line for any angle in the range tested. Consequently, $y_{11} < y_{\max}$ at all tested angles, and using $y_{\max}$ as the criterion overstates the risk of failure. Figure 2 illustrates the fundamental parameters.

Projectile Motion: Fundamental Parameters

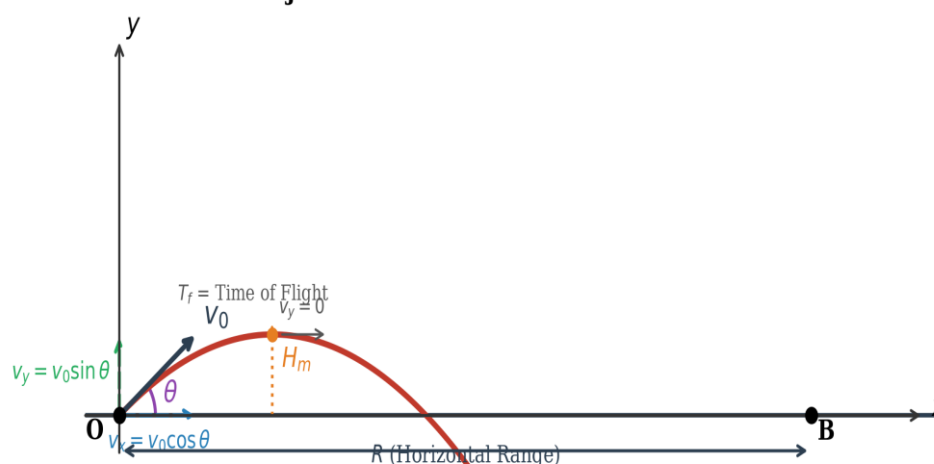


Figure 2. Projectile motion parameters. v_0 = initial velocity; θ = launch angle; v_x = horizontal component; v_y = vertical component; H_m = maximum height; R = horizontal range; T_f = total flight time. The ball has not yet reached H_m when it crosses the goal line at the penalty distance (11 m).

Phase I: Data Acquisition

A systematic review of peer-reviewed literature published between 2011 and 2024 was conducted to establish input parameters. Sources were included if they explicitly reported penalty kick kinematics, ball velocity measurements, or football physics parameters. Key values adopted were:

- Penalty kick distance: 11 metres (FIFA standard)
- Launch angles tested: $\theta \in \{10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ, 35^\circ, 40^\circ, 45^\circ\}$
- Fixed initial velocity: $v_0 = 24.3$ m/s (representative value from prior literature)
- Goalpost crossbar height: 2.44 metres (FIFA standard)
- Gravitational acceleration: $g = 9.81$ m/s²

The upper boundary of 45° was selected because it represents the theoretical maximum-range angle under idealised conditions (Ortega Contreras et al., 2018), providing a physically motivated ceiling. The goalkeeper's position was excluded to isolate the independent effect of launch angle on ball trajectory. Figure 3 illustrates the penalty kick scenario to scale.

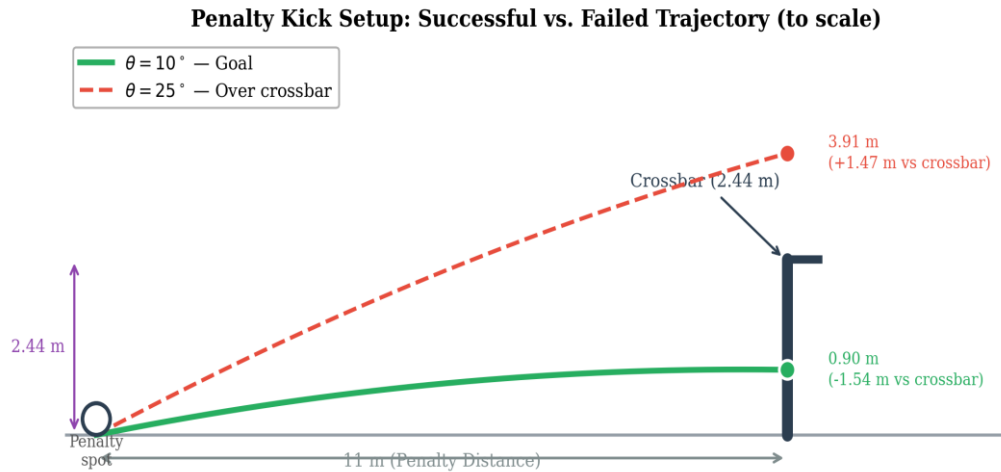


Figure 3. Penalty kick scenario drawn to scale. Green (solid): $\theta = 10^\circ$, ball height at goal line (y_{11}) = 0.90 m, margin = +1.54 m. Red (dashed): $\theta = 25^\circ$, y_{11} = 3.91 m, exceeds crossbar by 1.47 m. Dots mark y_{11} at $x = 11$ m, which is the operationally correct success criterion.

Phase II: Calculation

For each of the eight launch angles, the following five quantities were computed analytically using Equations (3.4)–(3.9):

- Time to maximum height: $t_{\max} = v_y / g$
- Maximum vertical displacement: $y_{\max} = v_y^2 / (2g)$
- Total time of flight: $T = 2v_y / g$
- Total horizontal range: $x = v_0^2 \sin(2\theta) / g$
- Ball height at goal line: $y_{11} = v_y(11/v_x) - \frac{1}{2}g(11/v_x)^2$

All results were verified independently through MATLAB numerical simulation, which produced values identical to four decimal places, confirming the analytical derivation. MATLAB generated the trajectory plots in the Results section.

Phase III: Evaluation Criteria

Goal-scoring success was defined by simultaneous satisfaction of two binary criteria derived from FIFA standard dimensions:

- Criterion 1 (vertical): $y_{11} < 2.44$ m [ball height at $x = 11$ m is below the crossbar]
- Criterion 2 (horizontal): $x > 11$ m [ball reaches or passes the goal line]

Criterion 1 uses y_{11} , not $y_{\max}$. This distinction is analytically significant: at 15° , $y_{\max} = 2.02$ m appears to provide a 0.42 m margin, but $y_{11} = 1.87$ m provides the actual margin of 0.57 m. At 20° , $y_{\max} = 3.52$ m suggests failure by 1.08 m above the crossbar, but $y_{11} = 2.87$ m shows the actual failure is 0.43 m. If either criterion was violated, the kick was classified as unsuccessful.

Results

Trajectory Parameters for Each Launch Angle

Table 1 presents the complete kinematic parameters for each launch angle, including both $y_{\max}$ and y_{11} as distinct columns. The critical boundary between success and failure lies between 15° and 20° . At all eight angles, the horizontal criterion is satisfied.

Table 1

Kinematic parameters of ball trajectories at eight launch angles ($v_0 = 24.3 \text{ m/s}$, $g = 9.81 \text{ m/s}^2$)

θ (°)	t_{max} (s)	y_{max} (m)	y at goal line (m)	x total (m)	Vertical criterion ($y_{11} < 2.44 \text{ m}$)	Horizontal criterion ($x > 11 \text{ m}$)	Goal scored
10	0.43	0.908	0.903	20.59	Satisfied	Satisfied	Yes
15	0.641	2.016	1.870	30.10	Satisfied	Satisfied	Yes
20	0.847	3.521	2.865	38.69	Violated	Satisfied	No
25	1.047	5.375	3.906	46.11	Violated	Satisfied	No
30	1.239	7.524	5.011	52.13	Violated	Satisfied	No
35	1.421	9.901	6.204	56.56	Violated	Satisfied	No
40	1.592	12.435	7.517	59.28	Violated	Satisfied	No
45	1.752	15.048	8.990	60.19	Violated	Satisfied	No

Note. t_{max} = time to maximum height; y_{max} = maximum vertical displacement (peak of trajectory); y at goal line = ball height when $x = 11 \text{ m}$ (the operationally correct criterion). Criterion 1 is evaluated against y at goal line, not y_{max} . All values verified in MATLAB.

Vertical Constraint Analysis

Figure 4 presents the vertical analysis in two panels. Panel (a) shows y_{11} (ball height at the goal line) and Panel (b) shows y_{max} (maximum trajectory height). Both panels classify the same two angles as successful (10° and 15°) and the same six as failed, but the absolute values and therefore the implied margins differ, with Panel (a) providing the operationally correct estimate.

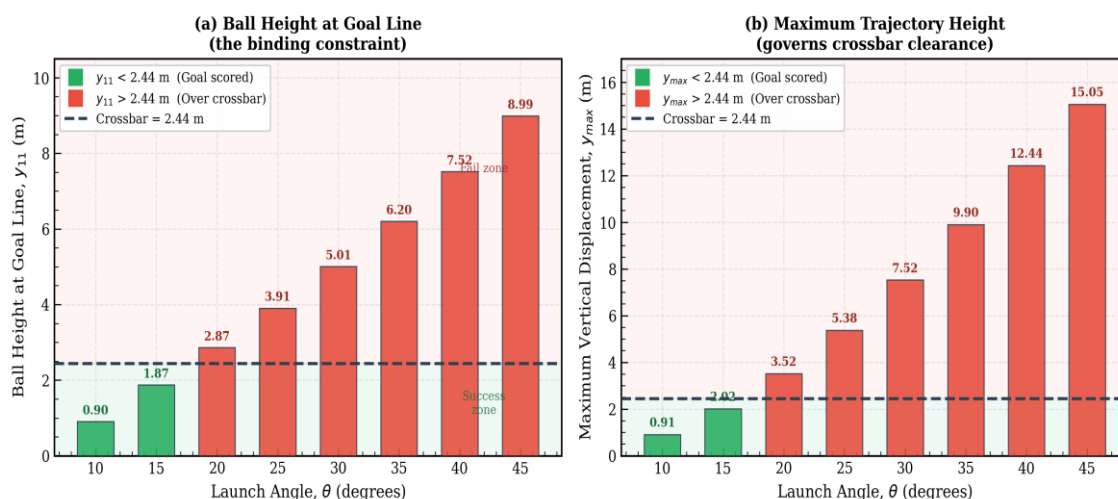
Vertical Constraint Analysis for Penalty Kick Trajectories ($v_0 = 24.3 \text{ m/s}$)

Figure 4. Vertical constraint analysis. Panel (a): ball height at goal line (y_{11}), which is the binding success criterion,. Panel (b): maximum trajectory height (y_{max}). At 15° , Panel (a) shows $y_{11} = 1.87 \text{ m}$ (margin = 0.57 m) while Panel (b) shows $y_{\text{max}} = 2.02 \text{ m}$ (apparent margin = 0.42 m). Values above bars are in metres.

At 10° , $y_{11} = 0.90 \text{ m}$ provides a 1.54 m margin below the crossbar, which is large enough to absorb substantial technical variation (an upward deviation of approximately 5° to 6° from target before failure). At 15° , $y_{11} = 1.87 \text{ m}$ leaves a 0.57 m margin, requiring tighter angular precision: a deviation of approximately 2° to 3° upward from a 15° target produces a failed trajectory. The transition from 15° to 20° increases y_{11} by 53.5 percent (from 1.87 m to 2.87

m), driven by the quadratic dependence of vertical displacement on the vertical velocity component (Equation 3.8). These results are consistent with Blair et al. (2020), who found empirically that lower kick-leg angles produce more accurate, lower-trajectory shots.

Horizontal Range: The Non-Binding Constraint

Table 2 reports horizontal ranges and their multiples of the 11 m penalty distance. Figure 5 plots the monotonic increase from 20.59 m at 10° to 60.19 m at 45°. Every tested angle exceeds the penalty distance by at least 87.2 percent (1.87× at 10°). The horizontal criterion is satisfied at all eight angles without exception.

Table 2

Horizontal range as a function of launch angle, with multiples of the penalty distance

θ (°)	Horizontal Range, x (m)	Multiple of 11 m	Horizontal Criterion
10	20.59	1.87x	Satisfied
15	30.10	2.74x	Satisfied
20	38.69	3.52x	Satisfied
25	46.11	4.19x	Satisfied
30	52.13	4.74x	Satisfied
35	56.56	5.14x	Satisfied
40	59.28	5.39x	Satisfied
45	60.19	5.47x	Satisfied

Note. At $v_0 = 24.3$ m/s, all angles generate horizontal ranges exceeding the penalty distance. The minimum multiple (1.87x at 10°) confirms that horizontal reach is not a binding constraint at this velocity. Range follows $x = v_0^2 \sin(2\theta)/g$, maximised at $\theta = 45^\circ$.

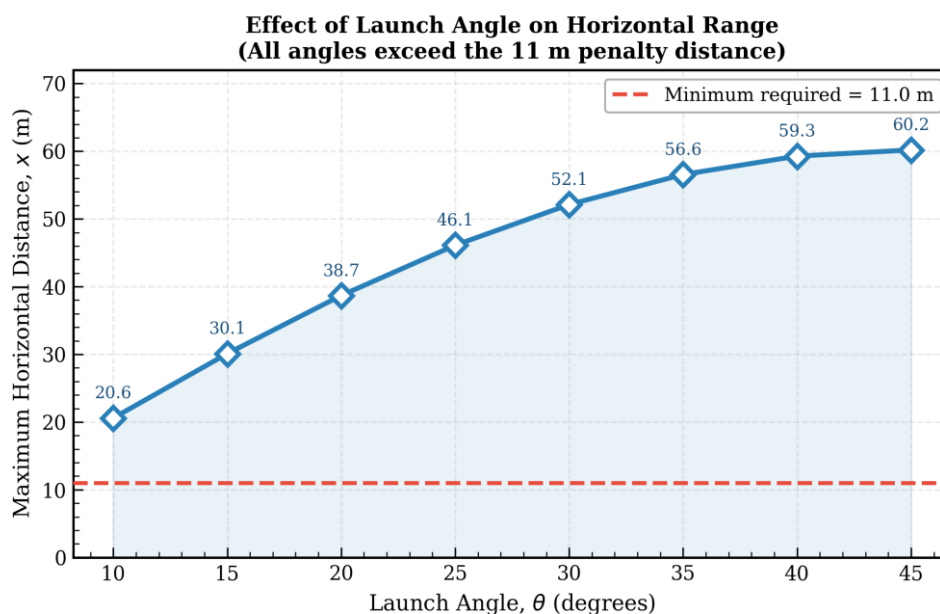


Figure 5. Horizontal range as a function of launch angle. All tested angles exceed the 11 m minimum (red dashed line). The curve follows $x = v_0^2 \sin(2\theta)/g$, as derived in Equation (3.6), and is maximised at $\theta = 45^\circ$.

The monotonic pattern follows directly from Equation (3.6) and is maximised at 45° as classical projectile theory predicts (Leela & Comissiong, 2011). The practical implication is

unambiguous: a kicker at $v_0 = 24.3$ m/s does not need additional force to reach the goal at any of the tested angles. Kicking power is a threshold variable, not an accuracy variable: angular precision in the vertical plane is the critical controllable parameter.

Trajectory Overlay and the Optimal Angular Window

Figure 6 presents all eight trajectories overlaid on a scaled goalpost. The 10° (green, solid) and 15° (orange, solid) trajectories are the only ones that remain below the 2.44 m crossbar when they cross the goal line. Dots on each trajectory mark y_{11} at $x = 11$ m, providing a direct visual comparison of the actual success criterion.

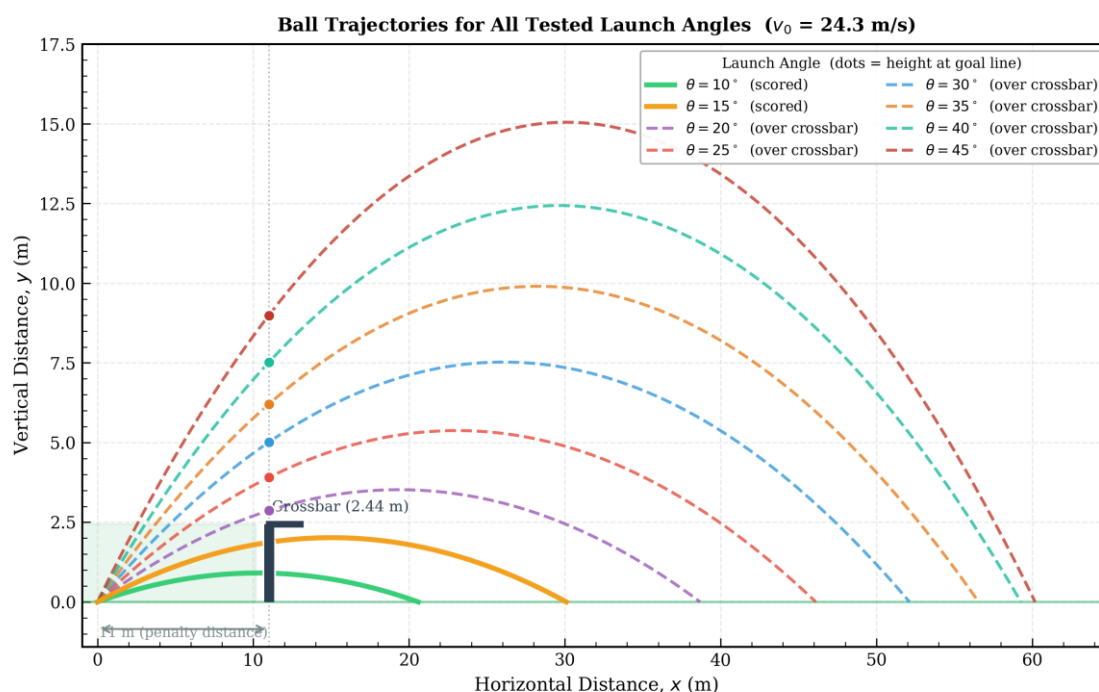


Figure 6. Ball trajectories for all eight tested angles ($v_0 = 24.3$ m/s). Solid lines: successful trajectories ($\theta = 10^\circ$ and 15°). Dashed lines: failed trajectories. Dots mark ball height at $x = 11$ m (y_{11}), which is the binding criterion. Goalpost ($h = 2.44$ m, $d = 11$ m) drawn to scale.

The overlay confirms the tight nature of the success window. Of eight systematically tested angles, only two produce successful trajectories. The 10° angle offers a 1.54 m margin at the goal line and robust tolerance for angular imprecision; the 15° angle offers a 0.57 m margin and requires greater angular consistency. All angles above 15° fail by margins ranging from 0.43 m (at 20°) to 6.55 m (at 45°) above the crossbar.

Discussion

The Vertical Constraint Is the Only Binding Limit

The central finding is that vertical displacement at the goal line, not horizontal range, is the sole binding constraint for penalty kick success at $v_0 = 24.3$ m/s. Every tested angle carries the ball past the goal line; what determines whether the kick succeeds is entirely vertical: whether the ball passes below 2.44 m at $x = 11$ m. This result is consistent with Nor-Al-Din et al. (2021), who found angular precision rather than velocity magnitude to be the critical variable for basketball free throws, and with Blair et al. (2020), who identified lower kick-leg angles as the kinematic predictor of accurate low-trajectory shots. The present study provides the quantitative basis for those findings in the specific context of the FIFA penalty kick: at standard penalty velocity, the critical controllable variable is angular control, not force production.

The implication for training is clear. Penalty practice programmes oriented primarily toward power development target the wrong variable. Power is a threshold variable: it is sufficient to send the ball 11 m at virtually any angle above approximately 5° , but it is not a precision variable. Angular control in the vertical plane is the precision variable, and training should be designed accordingly.

The y_{11} vs. $y_{\max}$ Distinction: Practical Significance

Within the scope of the reviewed literature, an analytical contribution of this study is the explicit separation of y_{11} (ball height at the goal line) from $y_{\max}$ (peak trajectory height). At $v_0 = 24.3$ m/s, the ball reaches its apex at a horizontal distance well beyond 11 m for all tested angles. At 10° , the total range is 20.59 m, so the apex sits at approximately 10.3 m horizontal distance, just before the goal line. At larger angles, the apex is further beyond the goal line. In all cases, $y_{11} < y_{\max}$.

At 15° , $y_{\max} = 2.02$ m suggests a margin of 0.42 m below the crossbar, but $y_{11} = 1.87$ m gives the correct margin of 0.57 m, which is 36 percent more generous than the peak-height estimate implies. A coach who gives the athlete feedback based on peak height underestimates the available margin at 15° . Conversely, at 20° , $y_{\max} = 3.52$ m suggests failure by 1.08 m, but $y_{11} = 2.87$ m shows the actual failure is 0.43 m. Using peak height overstates the degree of failure and may lead to greater technique correction than the situation warrants. Neither error is catastrophic in isolation, but it illustrates that y_{11} is the correct metric for coaching feedback.

Angular Tolerance and Training Design

Within the success window, the two viable angles offer substantially different technical tolerances. At 10° , the 1.54 m margin is large enough that deviations of approximately 5° to 6° upward from the intended trajectory remain within the goal frame. At 15° , the 0.57 m margin means deviations of approximately 2° to 3° above target produce failure. Under match-pressure conditions, where proprioceptive precision is reduced by arousal, fatigue, and time pressure, the 10° angle is substantially more robust than the 15° angle.

The 10° angle's robustness, however, comes with a tactical trade-off not captured by the physics model. An extremely flat trajectory arrives at the goal at a predictable and relatively low height (0.90 m), which is within the goalkeeper's reach without diving. The 15° angle arrives at 1.87 m, which is above the goalkeeper's comfortable reach but still well below the crossbar. In practice, the optimal coaching prescription is to target the lower half of the success window when angular consistency is uncertain, and to aim closer to 15° when the kicker's technique is consistent enough to exploit the tactical advantage of a higher goal-line height.

Limitations and Scope of the Idealised Model

This study adopts the idealised no-drag, no-spin model, which is valid as a baseline but brackets several real-world factors. Aerodynamic drag reduces effective range and alters trajectory shape, particularly at higher velocities; at the 11 m penalty distance, the effect is small but non-zero. Ball spin (the Magnus effect) introduces lateral and vertical deflection that can allow successful goals at angles above 15° if topspin or sidespin deflects the ball downward after the apex; Goff (2013) and Javorova and Ivanov (2018) both document this

effect. The fixed initial velocity assumption means results are specific to $v_0 = 24.3$ m/s; a sensitivity analysis varying v_0 would extend the findings to a broader range of kickers and conditions. Finally, the absence of goalkeeper modelling means the optimal angle from a pure physics perspective (10°) may not be optimal tactically. In addition, the novelty claims of this study are limited to the literature reviewed in Section 2, and it remains possible that studies outside the scope of this review have addressed the distinction between y_{11} and $y_{\max}$.

These limitations define a natural research agenda. Future work should incorporate aerodynamic drag and ball spin using computational fluid dynamics to test whether spin enables successful scoring above 15° ; parametrically vary v_0 to map the success window as a function of initial velocity; and integrate goalkeeper reach modelling to determine which angle within the success window minimises the probability of the shot being saved. Real-world validation using motion capture or high-speed video kinematic analysis would allow model predictions to be tested against measured launch angles and outcomes in trained players.

Conclusion

This study applied Newtonian mechanics and MATLAB simulation to systematically analyse penalty kick trajectories across eight launch angles (10° to 45°) at $v_0 = 24.3$ m/s. Three principal findings are reported. First, the vertical constraint (ball height at the goal line must not exceed 2.44 m at $x = 11$ m) is the sole binding criterion for penalty success at this velocity. The horizontal criterion ($x > 11$ m) was satisfied at all eight angles, with even the flattest trajectory (10°) carrying the ball to 20.59 m. Second, only launch angles of 10° and 15° satisfied the vertical criterion, establishing the optimal angular window as $0^\circ < \theta \leq 15^\circ$. Third, the angular safety margin is asymmetric within the success window: 10° provides a 1.54 m margin at the goal line (robust to angular variation) while 15° provides a 0.57 m margin (requiring greater precision). Within the scope of the reviewed literature, an analytical contribution of this study is the explicit distinction between $y_{\max}$ and y_{11} , which are analytically distinct quantities that produce different practical estimates of the available margin.

For coaches and performance analysts, the findings prescribe a training target (10° to 15° launch angle), identify angular control in the vertical plane as the critical precision variable, and quantify the risk asymmetry between the two successful angles. For researchers, the study demonstrates that computational physics modelling is a productive complement to empirical biomechanical measurement, capable of deriving prescriptive thresholds that field data alone cannot efficiently provide and of resolving analytical distinctions such as the y_{11} vs. $y_{\max}$ difference that matter for practical application.

Declaration

No human subjects were involved in this computational study. All data were derived from published literature and established physical constants. Ethics reference: REC/1093/2024.

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