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Causal Allocation and Hierarchical Virtual Metering for Product Carbon Footprint Accounting in Shared-Line Textile Manufacturing

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Abstract

In shared-line textile manufacturing, energy records, material transactions, and environmental data are often available only at the workshop or production-line level, while product carbon footprint accounting requires attribution to orders and batches. This study proposes a causal allocation and hierarchical virtual metering method that separates directly attributable quantities, shared resources, and unresolved common residuals. Physical measurements and business records define the actual total, while resource-specific causal drivers distribute only the shared portion. Materials are directly attributed where batch records are available; energy is allocated from the physical line meter to processes and then to orders or batches; and waste is assigned according to its generating mechanism. The method is tested with real production data from a cooperating ramie-textile enterprise by comparing virtual-meter estimates with temporary physical sub-meter measurements and simpler allocation rules. The results show that physically grounded drivers improve allocation accuracy while preserving measured totals. The proposed approach provides a practical and traceable basis for product carbon footprint accounting when permanent sub-metering is incomplete.

Keywords: Product Carbon Footprint, Shared Production Line, Causal Allocation, Virtual Metering, Textile Manufacturing

Introduction

Product carbon footprint (PCF) accounting requires a clearly defined functional unit, system boundary, and data-quality criteria, but it also depends on a defensible method for assigning life-cycle inventory activities to the product under study. Life cycle assessment (LCA) provides the methodological basis for such allocation. The International Organization for Standardization (ISO) specifies in ISO 14044:2006, Section 4.3.4, that allocation should first be avoided where feasible through subdivision or system expansion; when allocation cannot be avoided, inputs and outputs should be partitioned according to underlying physical relationships before other relationships are considered (ISO, 2006). ISO 14067:2018 applies these life-cycle principles to the quantification of product carbon footprints (ISO, 2018). Recent allocation reviews follow the same hierarchy and emphasize physically meaningful relationships when allocation is unavoidable (Kyttä, Roitto, Astaptsev, Saarinen, & Tuomisto, 2022; Dominguez Aldama, Grassauer, Zhu, Ardestani-Jaafari, & Pelletier, 2023). Research on product carbon-footprint practice also identifies activity-data quality and allocation choice as important sources of methodological uncertainty, particularly in textile and apparel systems (He, Wu, & Ding, 2025).

This problem is especially acute in textile plants where several orders share the same production lines. Degumming, dyeing, drying and heat-setting, finishing, and auxiliary systems may serve multiple orders or batches, while the underlying data are recorded at different operational levels. Enterprise Resource Planning (ERP) and Warehouse Management System (WMS) platforms capture orders, the bill of materials (BOM), material issues and returns, and inventory movements. Manufacturing Execution Systems (MES) record processes, equipment, batches, and production events. Programmable Logic Controllers (PLC), Supervisory Control and Data Acquisition (SCADA) systems, and smart meters provide equipment-state and energy-use data. The result is a persistent granularity gap: resource totals are measured at the enterprise, workshop, or line level, whereas PCF results must be reported at the order, batch, or functional-unit level.

Previous studies have examined parts of this problem from process and organizational perspectives. Manufacturing carbon-accounting models have traced product footprints across equipment, part, product, and workshop layers, showing why process-level differences should not be replaced by plant-wide averages (He, Qian, & Li, 2023). Xiang, Xu, Sah, Wei, and Fang (2024) examined the dynamic allocation of multilevel carbon data from manufacturing organizations to products and emphasized the need to map activity data across organizational and product levels. Nevertheless, incomplete sub-metering leaves two distinct questions. First, how much electricity, material, or waste occurred within the system boundary? Second, which products should bear the shared portion? Combining these questions in one estimation model can cause theoretical power to substitute for measured electricity, standard BOM demand to be treated as actual material use, or common residuals to be absorbed into product results merely to achieve numerical closure.

The method draws on three complementary perspectives. Empirical and case-based studies of Material Flow Cost Accounting (MFCA) emphasize transparent physical flows, resource-efficiency assessment, and the relationship between material losses and organizational performance; together, these provide a useful quantity-balance discipline (Walz & Guenther, 2021; Nishitani et al., 2022). Research on Activity-Based Costing (ABC) and Time-Driven Activity-Based Costing (TDABC) traces resource use through processes or activities to cost objects, with time-based representations offering a practical means of modeling heterogeneous resource demands (Sánchez-Rebull, Niñerola, & Hernández-Lara, 2023; Kefe & Tanış, 2023). This logic applies directly to energy attribution on shared production lines: equipment and production activities consume electricity, while orders and batches become responsible through their use of those activities. The allocation reviews in (Kyttä et al., 2022; Dominguez Aldama et al., 2023) define the methodological boundary by explaining when allocation should be avoided and why physically meaningful relationships are preferred when it remains necessary.

The study addresses three research questions. First, how can actual activity totals be separated from allocation weights under incomplete sub-metering so that product-level results remain anchored to physical meters or business records? Second, how should causal drivers differ for material, energy, and waste according to the mechanisms that produce these flows? Third, how can a two-stage virtual-metering method be formulated with analytical conservation properties and tested externally against physical sub-meters or historical measurements?

The study makes three contributions. It first introduces a unified activity-data attribution framework that separates direct attribution, shared-resource allocation, and common residuals while constraining the shared allocation to the observed total. The framework unifies the logic of attribution without imposing one driver formula on every resource. It then develops a two-stage energy model in which a physical line meter is disaggregated into process-level virtual meters before shared process electricity is attributed to orders or batches using production-event evidence. Finally, it formalizes conservation at both single and hierarchical levels, distinguishes arithmetic closure from physical metering accuracy, and empirically tests the energy-allocation component using real production-period line-meter, temporary-submeter, PLC, and MES data from a cooperating ramie-textile enterprise, with comparisons against simpler allocation methods.

Methodology

Research Object and Basic Assumptions

We consider a textile production system in which multiple orders and batches share common processes. The accounting hierarchy has four levels: production line, process, order, and batch. The batch is the smallest management object used for product-level activity attribution. The study period is divided into discrete windows defined by production events. Energy is attributed within each window and accumulated over the production cycle, so time-resolved operating evidence can support periodic PCF accounting without altering the model hierarchy. Four assumptions define the scope of the method. First, each resource has at least one credible source that establishes its actual total within the system boundary, such as a physical electricity or gas meter, WMS issue and return records, or an environmental ledger. Second, a quantity linked unambiguously to an order or batch through a document, sub-meter, or unique production event is attributed directly and bypasses the allocation model. Third, causal drivers determine only the relative shares of a common resource; they do not replace measured totals. Fourth, common activities that cannot be explained reliably with the available data remain explicit residuals rather than being eliminated through parameter tuning.

Unified Causal Allocation Model

Let r denote a resource category, let B_r denote the set of accounting objects associated with resource r, and let X_r denote the total activity quantity of resource r observed within the system boundary over the study period. X_r is decomposed into the amount $D_{r,b}$ directly attributable to object b, a shared-resource pool S_r that requires allocation, a common residual C_r, and an inventory or work-in-process change ΔI_r. The total-balance relationship is:

$$X_r = \sum_{b \in B_r} D_{r,b} + S_r + C_r + \Delta I_r \qquad (1)$$

where X_r is the actual total activity quantity of resource r; $D_{r,b}$ is the amount of resource r directly attributable to object b; S_r is the total shared quantity whose object-level ownership cannot be identified directly but for which an interpretable causal relationship exists; C_r is the common residual that cannot currently be explained with sufficient reliability; ΔI_r represents inventory, work-in-process, or cross-period carryover changes; and B_r is the set of accounting objects participating in the attribution of resource r.
For the shared-resource pool S_r, let $\varphi_{r,b} \geq 0$ be the causal driver for object b. When the sum of $\varphi_{r,i}$ over all i in B_r is greater than zero, the shared amount $A_{r,b}$ allocated to object b is:

$$A_{r,b} = S_r \frac{\phi_{r,b}}{\sum_{i \in B_r} \phi_{r,i}} \qquad (2)$$

where $A_{r,b}$ is the activity quantity of shared resource r allocated to object b; $\varphi_{r,b}$ is a causal driver representing the relative occupation or generation contribution of object b with respect to resource r; and i is an index over objects in the same shared-resource pool.
The attributable activity quantity received by object b at the current level is:

$$X_{r,b} = D_{r,b} + A_{r,b} \qquad (3)$$

where $X_{r,b}$ denotes the resource activity attributable to object b without forcing the common residual into product results. If the research objective requires C_r to be assigned ultimately to products, a separate secondary-allocation rule for common resources should be defined and disclosed explicitly rather than implicitly embedding C_r in the causal driver used in Eq. (2).
Equations (1)–(3) provide the common mathematical structure for all three flows. The resource-specific element is the definition of $\varphi_{r,b}$; the general principle remains unchanged: observed totals define the accounting boundary, while causal drivers determine how the shared portion is distributed.

Proposition 1 (single-level conservation of shared resources). If the shared-resource pool is allocated using the normalized non-negative causal drivers in Eq. (2), and the sum of the drivers is greater than zero, the sum of shared amounts allocated to all objects at that level is equal to the actual shared-resource total. This follows because the normalized weights sum to one. Hence, allocation changes only the attribution of the resource among objects and does not change the shared-resource total at that level.

Proposition 2 (hierarchical conservation in multilevel allocation). If every allocation layer—such as production line to process and process to order/batch—satisfies Proposition 1, and if direct-attribution amounts, common residuals, and inventory or work-in-process changes are retained explicitly, then the sum of activity attributable to terminal objects plus all retained terms necessarily reconciles to the actual activity quantity at the root node. Thus, the closure of hierarchical virtual metering arises from the recursive combination of locally conservative allocation steps rather than from ex-post balancing.

These propositions distinguish conservation from accuracy. The allocation rules prevent the observed total from being created or lost as data move through the hierarchy. The physical credibility of the resulting shares is a separate issue that requires calibration and external validation; algebraic closure alone cannot establish it.

Driver Selection and Allocation Priority

Driver selection follows one priority: physical interpretability takes precedence over statistical convenience. A candidate driver should have a causal relationship with resource use or waste generation, be observable in existing systems or stable engineering parameters, avoid redundancy with other drivers, remain sufficiently stable under comparable operating conditions, and be testable against sub-meter data, sampled measurements, single-process windows, or historical records.

Accordingly, the following allocation priority is adopted:

1. Direct attribution: when an activity quantity can be linked unambiguously to an order or batch through a business document or a unique production event, it is assigned directly without model-based allocation.
2. Causal allocation: only shared resources are allocated using drivers that represent the mechanism through which the resource is consumed or generated.
3. Common residual: activities such as common lighting, standby consumption, compressed-air use, or material retained in shared piping that cannot yet be explained reliably are retained explicitly as common residuals.
4. External calibration: model parameters are corrected using physical sub-meters, historical single-process windows, or sampled measurements rather than by tuning parameters solely to satisfy conservation.

This priority operationalizes the allocation hierarchy specified in ISO 14044:2006, Section 4.3.4, and is consistent with product carbon footprint quantification under ISO 14067:2018 (ISO, 2006, 2018). It is also supported by the allocation reviews of Kyttä et al. (2022) and Dominguez Aldama et al. (2023), which emphasize avoiding allocation where feasible and using physically meaningful relationships when allocation is unavoidable. The priority further reflects the activity-driver logic described in the ABC and TDABC literature (Sánchez-Rebull et al., 2023; Kefe & Tanış, 2023), where resource demand is traced through activities and time-based process drivers rather than assigned mechanically to output.

Material Flow Attribution

In a mature ERP/WMS environment, issue and return transactions, barcode records, or weighing data already link many major materials to production orders or batches. Material flows should therefore be attributed directly wherever possible. Model-based allocation is reserved for materials whose batch-level ownership remains unresolved, such as materials supplied from a common tank, a centralized liquid-feed system, or a shared dye bath.

$$D_{m,b} = I_{m,b} - R_{m,b} \qquad (4)$$

where $D_{m,b}$ is the amount of material m directly attributable to batch b; $I_{m,b}$ is the quantity issued to batch b; and $R_{m,b}$ is the quantity returned from batch b.

A standard-demand driver is introduced only for those genuinely shared materials. The driver represents the relative demand of each batch rather than an estimate of the total quantity consumed.

$$d_{m,b} = s_{m,p(b)} Q_b \kappa_{m,b} \qquad (5)$$

where $d_{m,b}$ is the standard-demand driver of batch b for shared material m; $s_{m,p(b)}$ is the standard unit consumption or recipe requirement of material m for product $p(b)$; $p(b)$ denotes the product associated with batch b; Q_b is the actual qualified output or processed quantity of batch b; and $\kappa_{m,b}$ is a loss or recipe-adjustment coefficient applied only when supported by process evidence, with a default value of 1 when such evidence is unavailable. The allocated amount of the shared material is then:

$$A_{m,b} = S_m \frac{d_{m,b}}{\sum_i d_{m,i}} \qquad (6)$$

where $A_{m,b}$ is the amount of shared material m allocated to batch b; S_m is the actual total consumption of the shared material; and i indexes the batches participating in allocation of that shared material.

This distinction prevents standard demand from being confused with actual consumption. BOM or recipe information supplies relative weights for the unresolved shared portion, while enterprise records establish the quantity to be allocated. The approach preserves the physical-quantity discipline described in MFCA studies (Walz & Guenther, 2021; Nishitani et al., 2022).

Energy flow, Stage 1: From the Physical Line Meter to Process-Level Virtual Meters

Energy provides the clearest example of a measured total without sufficiently detailed product ownership. A Virtual Metering Point (VMP) is a digital estimate of local energy use applied when a permanent physical sub-meter is unavailable. Sossenheimer, Vetter, Stahl, Weyand, and Weigold (2021) found that VMPs can improve near-real-time transparency at the equipment level, although the estimates still require calibration or validation against measurements. The present method uses virtual metering for a different purpose. It does not estimate the plant total; the physical meter establishes that total, and virtual meters attribute the shared portion to lower-level objects.

Let E_t be the actual electricity recorded by the physical production-line meter within time window t. Let E_t^D denote electricity that can be directly attributed using an existing sub-meter or a unique event, and let E_t^C denote common auxiliary or currently unexplained electricity. The shared electricity entering process-level virtual metering is:

$$E_t^S = E_t - E_t^D - E_t^C \qquad (7)$$

where E_t is the physical line-meter electricity in time window t; E_t^D is directly attributable electricity; E_t^C is common residual electricity; and E_t^S is the shared electricity that must be allocated among processes through virtual metering.

When a reliable actual power curve is available for equipment e in process j, the preferred driver is the energy obtained by integrating that measured power over the time window:

$$q_{e,t} = \int_{t_0}^{t_1} P_e^{\mathrm{real}}(\tau)\, d\tau \qquad (8)$$

where $q_{e,t}$ is the energy driver for equipment e in time window t; $P_e^{\mathrm{real}}(\tau)$ is the actual power of equipment e at time τ; and t_0 and t_1 are the start and end times of the window, respectively.

If no reliable power curve is available, the equipment contribution is approximated with an engineering proxy:

$$\hat{q}_{e,t} = P_e^{\mathrm{rated}} \tau_{e,t} \lambda_{e,t} \kappa_{e,t} \gamma_e \qquad (9)$$

where $q_{e,t}$ is the proxy energy driver for equipment e in time window t; P_e^{rated} is rated equipment power; $\tau_{e,t}$ is effective operating time; $\lambda_{e,t}$ is the load factor; $\kappa_{e,t}$ is the process-energy-intensity adjustment; and γ_e is an equipment calibration coefficient obtained from

historical measurements or temporary sub-metering. If $P_e^{\text{real}}(\tau)$ already captures load and process-state effects, λ or κ should not be applied again, so as to avoid double adjustment.
The equipment-level contributions are then aggregated within each process. Let Ω_j denote the equipment set associated with process j; the process energy driver is:

$$v_{j,t} = \sum_{e \in \Omega_j} \tilde{q}_{e,t} \qquad (10)$$

where $v_{j,t}$ is the energy driver of process j in time window t; $q_{e,t}$ is selected according to data availability from either the measured-power or engineering-proxy formulation; and Ω_j is the equipment set associated with process j.
The process-level virtual-meter value is obtained by normalizing these process drivers against the shared electricity that remains to be allocated:

$$E_{j,t}^{V} = E_t^{S} \frac{v_{j,t}}{\sum_{k \in J_t} v_{k,t}} \qquad (11)$$

where $E_{j,t}^{V}$ is the virtual electricity allocated to process j within time window t; J_t is the set of processes participating in allocation of the shared electricity in time window t; and k is the process index.

Stage 1 can therefore be read in operational terms as follows: the physical meter fixes how much electricity actually entered the allocation problem, while equipment-operation data determine how that shared amount is split among processes. A mismatch between proxy drivers and the line-meter total does not alter the measured total; it changes only the relative attribution shares and should be addressed through calibration.

Energy flow, Stage 2: from process-level virtual meters to order/batch-level virtual meters
The second stage answers a different question from the first. Stage 1 identifies where shared electricity is consumed and is therefore centered on equipment and processes. Stage 2 asks which orders or batches should bear the process-level electricity and therefore relies on production events and task occupation. Keeping these questions separate prevents a line-level meter from being allocated directly to products on the basis of production volume alone. Where an MES event shows that a particular equipment interval serves only one batch, the associated electricity is attributed directly and no second-stage allocation is needed. A batch-level task driver is introduced only when auxiliary systems are shared, several tasks draw from the same process pool, or production-event boundaries are not fine enough to identify ownership directly:

$$g_{b,j,t} = \sum_{e \in \Omega_j} P_e^{\text{rated}} \tau_{b,e,t} \lambda_{b,e,t} \kappa_{b,e,t} \eta_{b,e,t} \qquad (12)$$

where $g_{b,j,t}$ is the task-level energy driver of batch b in process j and time window t; P_e^{rated} is rated power of equipment e; $\tau_{b,e,t}$ is the effective time for which batch b occupies equipment e; $\lambda_{b,e,t}$ is the load factor during operation of that batch; $\kappa_{b,e,t}$ is the energy-intensity adjustment for the corresponding process recipe; $\eta_{b,e,t}$ is a validated task adjustment such as loading ratio or standard work quantity; and Ω_j is the equipment set associated with process j.

For plant implementation, the task driver can be simplified when a stable historical process-intensity library is available. In that case, the standard energy demand of a batch is calculated as its actual processed quantity multiplied by the historical unit energy intensity for the same or most comparable process route. The resulting standard-demand values determine only the relative batch shares of the process-level electricity pool; they do not replace the process-level virtual-meter total. This transparent form is used in the observed Stage 2 field example reported below.

If $E^S_{j,t}$ denotes the process-level electricity that still requires allocation within time window t, the corresponding batch-level virtual electricity is calculated from the normalized task drivers:

$$E^V_{b,j,t} = E^S_{j,t} \frac{g_{b,j,t}}{\sum_{i\in B_{j,t}} g_{i,j,t}} \qquad (13)$$

where $E^V_{b,j,t}$ is the virtual electricity of process j allocated to batch b; $B_{j,t}$ is the set of batches occupying process j within time window t; and i is the batch index.
The electricity attributable to a batch over the full study period is obtained by accumulating both directly attributable intervals and second-stage virtual allocations:

$$E_b = \sum_t \sum_j \left(E^D_{b,j,t} + E^V_{b,j,t}\right) \qquad (14)$$

where E_b is the cumulative electricity attributable to batch b; $E^D_{b,j,t}$ is electricity directly attributed through a unique equipment event or physical sub-meter; and $E^V_{b,j,t}$ is shared electricity obtained through the second-stage virtual-metering model.
Equations (7)–(14) thus define a hierarchical chain from a physical line meter to process-level virtual meters and, where necessary, to order- or batch-level virtual meters. The term virtual refers only to the absence of a permanent meter at the lower level; the root quantity remains anchored to physical measurement.

Waste Flow Attribution Based on Generation Mechanisms
Waste flows differ from material and energy inputs because they are generated as outputs of production. Their drivers should therefore reflect the mechanisms by which waste or environmental loads arise. Direct attribution is preferred when an independent outlet, online monitor, weighing record, or process ledger identifies the source. Only the unresolved shared portion is allocated using an environmental-load driver:

$$h_{w,b} = Q_b a_{w,p(b)} s_{w,b} \qquad (15)$$

where $h_{w,b}$ is the environmental-load driver of batch b for waste stream w; Q_b is the actual processed quantity of batch b; $a_{w,p(b)}$ is the waste- or pollutant-generation coefficient per unit of product $p(b)$; and $s_{w,b}$ is a process adjustment reflecting factors such as the number of washing cycles, dye-bath intensity, pollutant load, or other mechanisms relevant to waste generation.
The shared waste stream is then distributed across batches according to the normalized environmental-load drivers:

$$A_{w,b} = S_w \frac{h_{w,b}}{\sum_i h_{w,i}} \qquad (16)$$

where $A_{w,b}$ is the amount of shared waste stream w allocated to batch b; S_w is the actual total quantity of the shared waste stream; and i indexes the batches participating in the allocation.

Environmental activity indicators such as wastewater volume, sludge mass, or chemical oxygen demand are not greenhouse-gas emissions in themselves. They enter the PCF calculation only when the study boundary and the relevant treatment or disposal emission factors support conversion to carbon dioxide equivalent (CO2e). Care is also needed to avoid double counting, for example by including both fuel combustion activity and the same combustion emissions as separate sources.

Integration of the three Flows and Product Carbon Footprint Calculation

Once material, energy, and waste-treatment activities have been attributed to batches, their greenhouse-gas contributions can be combined to calculate batch-level PCF:

$$CF_b = \sum_m M_{m,b}\, EF_m + \sum_e E_{e,b}\, EF_e + \sum_w W_{w,b}\, EF_w \qquad (17)$$

where CF_b is the carbon footprint of batch b, typically expressed in kg carbon dioxide equivalent (CO2e); $M_{m,b}$ is the activity quantity of material m attributed to batch b; $E_{e,b}$ is the activity quantity of energy type e attributed to batch b; $W_{w,b}$ is the waste-treatment activity attributed to batch b; and EF_m, EF_e, and EF_w are the emission factors for the corresponding material, energy, and waste-treatment activities, respectively. The temporal, geographic, technological, and source representativeness of emission factors should be consistent with the study objective and evaluated separately in a formal PCF study.

If batch b contains N_b functional units, the corresponding carbon footprint per functional unit is:

$$PCF_b = \frac{CF_b}{N_b} \qquad (18)$$

where PCF_b is the carbon footprint per functional unit for batch b and N_b is the number of functional units in that batch. PCFs based on different functional units should not be ranked directly in terms of environmental performance unless functional equivalence has been demonstrated.

Balance Checks, External Validation, and Robustness Evaluation

For any resource r, define the balance difference as:

$$\Delta_r = X_r - \left(\sum_b D_{r,b} + \sum_b A_{r,b} + C_r + \Delta I_r\right) \qquad (19)$$

where Δ_r is the balance difference for resource r; the remaining symbols are defined in Eqs. (1) and (2). The relative balance error is defined as:

$$\varepsilon_r^{\mathrm{bal}} = \frac{|\Delta_r|}{X_r} \qquad (20)$$

The balance error is a bookkeeping test. It can reveal omissions, duplication, and sign errors in the attribution chain, but it cannot show whether the selected drivers reproduce physical consumption accurately.

Virtual meters should therefore be tested against physical sub-meters on key equipment or against temporary high-accuracy measurements. Let y_n denote the measured electricity in validation window n, let $\hat{y}_n$ denote the corresponding virtual-meter estimate, and let N be the number of validation windows. The Mean Absolute Error (MAE) is:

$$MAE = \frac{1}{N}\sum_{n=1}^{N} |\hat{y}_n - y_n| \qquad (21)$$

The Mean Absolute Percentage Error (MAPE) is:

$$MAPE = \frac{100\%}{N} \sum_{n=1}^{N} \left| \frac{\hat{y}_n - y_n}{y_n} \right| \qquad (22)$$

where y_n is the measured electricity in validation window n, $\hat{y}_n$ is the corresponding virtual-metering estimate, and N is the number of validation samples. MAPE should not be used directly when y_n is close to zero; more robust error measures should be considered in such cases.

Robustness must be tested rather than assumed. Load factors, process-intensity adjustments, standard-demand corrections, and related parameters can be perturbed to measure the sensitivity of product-level results. The method should also be compared with simpler benchmarks, such as production-volume or runtime-only allocation. Additional causal detail is warranted only when it improves agreement with external measurements or stability across operating conditions.

Empirical Field-Validation Design

To evaluate practical performance, anonymized real production-period data were obtained from a cooperating ramie-textile enterprise over 15 consecutive days. The dataset contains 1,440 production-line observations at 15-minute intervals and 4,320 matched equipment records for three representative energy-intensive processes: degumming (120 kW), dyeing (200 kW), and drying and heat-setting (180 kW). Production-line meter readings, temporary physical sub-meter measurements, and corresponding Programmable Logic Controller and Manufacturing Execution System records were time-aligned. After data-quality screening, 596 windows in which the three processes were simultaneously in valid production status were retained; the first 417 windows were used for calibration and the final 179 windows formed an independent time-ordered test set. Enterprise-identifying information was removed for commercial confidentiality.

The temporary physical sub-meters served as the external reference for evaluating the virtual-meter estimates. Five allocation rules were compared: equal allocation, runtime-only allocation, rated power multiplied by runtime, rated power multiplied by runtime and load factor, and a calibrated version of the power-runtime-load rule. Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) were used to quantify the difference between virtual and physical measurements. A separate observed shared-process record from the same enterprise was used to illustrate the second-stage allocation from a process virtual meter to concurrent batches.

Results and Validation

Analytical property 1: separation of total-quantity boundaries and attribution weights

Equation (2) and Proposition 1 show that total measurement and attribution weights are mathematically separable. A physical meter, issue and return records, or an environmental ledger fixes the shared-resource total; the causal driver determines only the relative shares assigned to individual objects. Calibration can therefore change the distribution of the shared quantity without changing the activity observed within the system boundary. For energy, a revised driver may alter the process split, but it cannot change the electricity recorded by the line meter.

Analytical property 2: hierarchical closure of two-stage virtual metering

Proposition 2 applies the same logic across levels. If allocation from the line meter to process-level virtual meters conserves the shared quantity, and allocation from each process pool to orders or batches conserves its local total, then terminal batch allocations reconcile to the root physical meter once direct quantities and retained residuals are included. Hierarchical virtual metering can therefore bridge the granularity gap without redefining the metering boundary.

This property also dictates how incomplete evidence should be handled. If available process drivers cannot account for all line-level electricity, the unexplained amount remains a common residual rather than being forced into a process weight. Likewise, if second-stage production events are insufficiently reliable, the corresponding quantity should remain at the process or common level until stronger evidence is available.

Validation framework: complementarity of four types of evidence

Validation requires several forms of evidence because no single test can establish both bookkeeping integrity and physical accuracy. Four tests are proposed: algebraic closure to confirm that quantities are neither lost nor duplicated; direct-attribution verification against ERP/WMS records and MES events; external validation of virtual meters against physical sub-meters or temporary measurements using MAE, MAPE, and systematic bias; and robustness testing against simpler rules such as production-volume or runtime-only allocation.

Each test answers a different question. Algebraic closure tests arithmetic integrity. Business-record verification checks whether activities are mapped to the correct objects. Physical comparison evaluates virtual-meter accuracy, while benchmark comparison determines whether the additional model complexity improves on simpler allocation rules.

Empirical Validation Results

The 179-window independent test set shows a clear difference between simple allocation rules and physically grounded virtual-meter drivers. Equal allocation and runtime-only allocation produced Mean Absolute Percentage Error values of 26.00% and 26.06%, respectively. Including rated power reduced the error to 6.01%, and adding observed load reduced it further to 3.18%. The calibrated power-runtime-load method achieved the best overall agreement with the temporary physical sub-meters, with a Mean Absolute Error of 1.10 kWh per 15-minute equipment window, a Mean Absolute Percentage Error of 3.03%, and a mean allocation-share error of 1.07 percentage points. In comparison, the corresponding share error under equal allocation was 7.39 percentage points. The results indicate that equipment power and actual load explain a substantial part of the between-process difference that is missed by simple equal or time-based allocation.

Table 1
Independent test-set comparison with temporary physical sub-meter measurements

Method	MAE (kWh/15 min)	MAPE	Mean share error (percentage points)
Equal allocation	7.62	26.00%	7.39
Runtime only	7.63	26.06%	7.41
Rated power × runtime	1.93	6.01%	1.87
Rated power × runtime × load	1.14	3.18%	1.11
Calibrated power × runtime × load	1.10	3.03%	1.07

All methods allocate the same measured electricity pool and therefore preserve the observed total. Their difference lies in how closely the allocated shares agree with the temporary physical sub-meters. The field results therefore support the central distinction of this study: conservation is necessary, but physical accuracy must be established through external measurement. A representative independent test window provides a more direct illustration.

Representative 15-minute Validation Window

In one independent production window, the three temporary physical sub-meters recorded a combined 102.698 kWh. The measured values were 21.723 kWh for degumming, 42.047 kWh for dyeing, and 38.928 kWh for drying and heat-setting. Equal allocation assigned 34.233 kWh to each process, whereas the calibrated power-runtime-load method allocated 21.921, 43.463, and 37.315 kWh, respectively. Both approaches preserve the same 102.698 kWh total, but the causal virtual-meter result is much closer to the physical distribution.

Table 2
Representative comparison for one observed 15-minute production window

Process	Physical sub-meter (kWh)	Equal allocation (kWh)	Calibrated virtual meter (kWh)
Degumming	21.723	34.233	21.921
Dyeing	42.047	34.233	43.463
Drying/heat-setting	38.928	34.233	37.315
Total	102.698	102.698	102.698

Process-to-Batch Field Illustration

A separate observed shared-process record was used to illustrate the second stage of the hierarchy. Stage 1 attributed 296.161 kWh to a shared dyeing and heat-setting process. Two concurrent batches occupied that process. Batch N001 processed 712.276 kg and had a historical same-process unit energy intensity of 0.30 kWh/kg, giving a standard energy-demand weight of 213.683. Batch C001 processed 146.642 kg with a corresponding historical intensity of 0.34 kWh/kg, giving a weight of 49.858. After normalization, the shares were 81.08% and 18.92%, and the process electricity was allocated as 240.132 kWh to N001 and 56.030 kWh to C001. These standard energy-demand values are allocation weights, not measured electricity; the 296.161 kWh process total remains fixed by Stage 1. Because no independent batch-specific electricity meter was available, this example demonstrates operational traceability and hierarchical conservation rather than independent batch-level metering accuracy.

Discussion

Theoretical Contribution

The main methodological contribution is the separation of quantity measurement from product-level attribution. PCF calculations are commonly expressed as activity data multiplied by emission factors, but shared-line manufacturing introduces an earlier problem: activities are observed at a coarser level than the required product result. The framework uses physical meters and business records to establish totals and causal drivers to divide only the shared portion. This allows the validity of the total and the reasonableness of the allocation to be examined independently.

The framework does not replace established LCA allocation hierarchies. ISO 14044:2006, Section 4.3.4, specifies a stepwise allocation procedure in which allocation should first be avoided where possible, and, where unavoidable, inputs and outputs should be partitioned using underlying physical relationships before other relationships are considered; ISO 14067:2018 applies LCA principles to product carbon footprint quantification (ISO, 2006, 2018). The reviews in Kyttä et al. (2022) and Dominguez Aldama et al. (2023) apply the same hierarchy. The present framework instead provides an operational method for resolving data-granularity mismatches at the manufacturing-execution level. Activities are attributed directly when possible, meaningful drivers are used for unavoidable allocation, and quantities without a stable causal relationship remain common residuals. The method is therefore an activity-data attribution mechanism within established PCF practice, not a new LCA allocation standard.

Relationship with Material Flow Cost Accounting and Activity-Based Costing Methods

Within the framework, Material Flow Cost Accounting (MFCA) contributes a physical-quantity discipline rather than a ready-made formula for product allocation. Empirical and case-based MFCA studies emphasize the visibility of material flows, losses, and resource-efficiency effects, which supports the rule that observed quantities define the total while standard demand is used only to divide the unresolved shared portion (Walz & Guenther, 2021; Nishitani et al., 2022). Thus, when Enterprise Resource Planning (ERP) and Warehouse Management System (WMS) records already identify batch-level material issues and returns, those values are attributed directly; the allocation model is reserved for common tanks, centralized liquid-supply systems, shared dye baths, and similar situations in which batch ownership is not directly observed.

Activity-Based Costing (ABC) and Time-Driven Activity-Based Costing (TDABC) contribute a complementary causal perspective. The systematic and manufacturing-oriented studies cited in (Sánchez-Rebull et al., 2023; Kefe & Tanış, 2023) show that ABC remains applicable to manufacturing settings and that TDABC represents heterogeneous resource requirements through activity- and time-based process drivers, including capacity-related information. The present method does not treat cost drivers as carbon drivers by default. Rather, it borrows the resource-activity-object logic and uses equipment power, operating time, load, and process state as candidate energy drivers that can be calibrated against measurements.
The three theoretical foundations have different roles. LCA and PCF standards define when allocation should be avoided and which relationships are acceptable. MFCA reinforces the physical closure of activity quantities. ABC and TDABC supply the causal structure linking shared resources to activities and then to accounting objects. The resulting operational

sequence is straightforward: establish the boundary from observed data, allocate only the shared portion with a mechanism-based driver, and report unresolved residuals explicitly.

Incremental Value and Applicability Limits of Two-Stage Virtual Metering

Research on Virtual Metering Points (VMPs) has largely examined equipment-level energy transparency where independent physical meters are unavailable (Sossenheimer et al., 2021). This study embeds virtual metering in a hierarchical attribution chain for product carbon accounting. Stage 1 uses the physical line meter as the electricity boundary and derives process-level virtual meters from process energy drivers. Stage 2 uses MES production events and task-occupation relationships to attribute shared process electricity to orders or batches. The two stages separate where electricity is consumed from which production object should bear it, thereby avoiding direct allocation from the line meter to products based only on production volume.

The approach has clear limits. Virtual meters compensate for missing metering granularity but are not equivalent to physical sub-meters. Their accuracy depends on power signals, time synchronization, equipment states, load factors, process parameters, and production-event records. Reliable sub-meter readings should take precedence and should calibrate or validate the virtual model. When key equipment or event data are missing, attribution should stop at the highest level supported by the evidence rather than being forced to the product level because the equations allow it.

Managerial and Research Implications

For managers, stable batch-level activity data can turn PCF from a period-end aggregate into a diagnostic measure. Process and batch virtual meters can show whether energy differences are associated with equipment occupation, load, or energy-intensive operating conditions. The enterprise validation further shows that physically grounded drivers can improve the credibility of energy attribution when permanent sub-metering is incomplete. These interpretations remain meaningful only when the allocation model is checked against relevant business records or physical measurements.

For researchers, the framework is deliberately falsifiable. Causal drivers are not validated merely because the model closes arithmetically; they must be tested using physical sub-meter data, temporary measurements, and alternative allocation rules. If a more elaborate driver does not reduce external error or improve stability across operating conditions, it should be simplified. Observable validation gains, rather than computational detail, must justify model complexity.

Study Scope and Further Validation

The empirical evidence in this study comes from a 15-day production period at one cooperating ramie-textile enterprise. Temporary physical sub-meter data provide independent accuracy evidence for the first stage of virtual metering, while the second stage is illustrated with an observed shared-process production record because batch-specific physical electricity meters were not available. These limits should be considered when interpreting the findings. Future research should extend validation across longer production periods, additional equipment, product mixes, operating conditions, and manufacturing plants. Product carbon footprint accuracy also depends on emission-factor quality and system-boundary choices, so the proposed activity-data attribution method does not replace a complete life cycle assessment data-quality evaluation.

Conclusion

This study proposes a product carbon footprint accounting method for shared-line textile manufacturing that combines causal allocation with hierarchical virtual metering. It separates activity data into directly attributable quantities, shared resources, and common residuals. Physical measurements and business records establish actual totals, while mechanism-based causal drivers distribute only the shared portion. The method therefore keeps total-quantity measurement separate from the judgment involved in attributing shared activity.

Materials are assigned directly from batch-level ERP/WMS issue and return records wherever possible. Only genuinely shared materials, including those supplied through common tanks or centralized systems, are allocated using standard-demand weights derived from BOM data, actual output, and justified corrections. Energy attribution proceeds in two stages. The first divides a physical line-meter total among process-level virtual meters using measured equipment power or an engineering proxy based on rated power, operating time, load, and process conditions. The second uses MES production events and batch task characteristics to distribute shared process electricity to orders and batches. Waste follows the same general attribution framework but uses drivers tied to the processes and environmental-load mechanisms that generate it.

The conservation properties show that allocation does not alter the observed total when each layer uses normalized drivers and retains direct quantities, common residuals, and inventory or work-in-process changes. The enterprise validation further shows that the physically grounded virtual-meter drivers reproduce temporary physical sub-meter measurements more closely than simpler allocation rules. The observed process-to-batch case also demonstrates how a process-level electricity pool can be transparently attributed to concurrent batches without redefining the measured quantity boundary.

The framework treats PCF accounting in shared manufacturing as verifiable activity-data attribution under an observed-total constraint. The cooperating-enterprise validation supports the practical applicability of the proposed energy-allocation logic under the studied production conditions, while broader external validity remains to be established. Future research should extend the validation across longer periods, more equipment and product mixes, exceptional operating events, and additional manufacturing plants, with continued calibration against independent physical measurements where feasible.

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