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Newman's Error Analysis of Common Errors in Solving Convergence of Power Series Using Ratio Test

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Abstract
In mathematics, the students are urged to answer the questions correctly. Answers with complete sets of solutions shows a certain level of understanding of students. However, it is undeniable that some student had difficulty in answering the questions correctly. The students may not have certain understanding on a particular topic and that does not mean that they are poor in mathematics. Some errors that students do in doing mathematics may due to misunderstanding of questions, incorrect concepts, careless mistakes or skip of required answer steps. The purpose of this study was to give insight to the instructors on the common errors done by the students in solving question with long sets of solution. This research method is a descriptive study, with the aim of finding out the number of percentage and the level of students' mistakes using Newman's Error Analysis. This study focused on year two student that undertook Further Calculus in Engineering emphasised on convergent test of power series using ratio test topic. The data were collected from their final examination answer papers, focused only on related questions. The results show the most common error made by the students were transformation error (38%) and encoding error (38%) and did less in comprehension error (2%). While reading error (5%) and process skill error (17%) could also had been considered low. Instructors must guide the students more on correct transformation (solve fraction and factorisation) and encoding (interval of convergence) in order to solve convergence of power series using ratio test.
Keywords: Convergence Series, Mathematics Error, Newman's Error Analysis, Power Series, Ratio Test.

Introduction

Instructors always wanted the best for their students, that is why there are various type of methods for teaching and learning. In mathematics, instructors always guided the students on understanding the problem and the methods to solve it until they got the correct solution. Students often relied on instructors for examples while practicing their exercise. However, in recent years, analysis of errors in solving mathematics problem have been practiced in order for the students to self-conduct their error analysis (Karnasih, 2015; Kristianto & Saputro, 2019). Combining the examples from the instructors and error analysis by both instructors and students, it helped in better understanding in mathematics problem solving.

In teaching and learning mathematics, problem solving is an important practice. Students must go through numerous steps before providing a solution because the problem-solving process is complicated. According to Polya (1973), problem solving have four stages which are understanding the problem, planning the strategy, implementing the plan, and reviewing the answers. For the first principle, students must learn the necessary underlying mathematical concepts and consider the terminology and notation used in the problem. Planning strategy need students to choose an appropriate strategy and try to solve it. For examples, draw a picture, look for a pattern or make a list. For the third principle, once students have an idea for a new approach, jot it down immediately. If the plan does not seem to be working, then students should start over and try another approach. Student must keep trying until something works. Last principle is look back where students need to find a potential solution, check to see if it works by answering some question, i.e., “Did you answer the question?” or “Is your result reasonable?”. If yes, students may proceed with the conclusion otherwise students have to repeat second and third principles.

There are several researches that used Polya’s method to analyze error analysis among students. For example, Siregar (2018) used Polya’s four steps approach to analyse the types of students’ error in solving pedagogic problems. Sulistyorini (2018) also used the same approach in error analysis solving geometry problem, where the results show that error occurred because of the misunderstanding of line segment’s measure that led to failure to understand triangles. Polya steps method also had been used to explain the limitations that high school students face when drawing logarithmic graphs. As a result, respondents' difficulties in identifying the issue and devising a solution were based on Polya measures (Agustina et al., 2019).

Other subject like algebraic and prism or pyramid problems had also been analysed to identified frequent error done by the students (Son et al., 2019; Hasanah & Yulianti, 2020). Polya steps method also had been used in mathematics problems based on storey questions. According to the findings, some students still have trouble solving mathematical problems in the form of storey problems (Maulyda et al., 2020). Teachers also being evaluated for their problem-solving ability namely identifying information on the problem, carrying out the procedure according to plan, and doing calculations correctly (Maulyda et al., 2019, Yayuk & Husamah, 2020).

Tambychika and Meerahb (2010) studied the difficulties of the students in solving mathematical problems. As a result, many mathematics skills, such as number-fact, visual-spatial, and information abilities, were lacking among the responders. The ability to get information was the most critical. Mathematical problem solving was constrained by a lack of mathematical skills and cognitive capacities in learning. Mayer (1992) considers two stages in solving the word problems such as problem comprehension and representation and searching for the solution and its implementation. He allocated special knowledge for each stage such as linguistic knowledge, comprehension knowledge, communicational knowledge, and calculation knowledge.

Newman’s Error Analysis (NEA) also being used to analyse students’ errors in solving mathematical problems. NEA classification errors contains reading, comprehension, transformation, process skill and encoding errors. Newman's error analysis (NEA) emerged from 1970s study on mathematics language difficulties. According to the hypothesis, a student goes through five stages when solving a word problem in mathematics. Reading and decoding is the first stage, in which students must read the problem and decode words and symbols. The understanding stage is the second step. At this point, students should be able to make sense of what they have read. The third stage is transformation, in which students must ‘mathematise' the problem, i.e., figure out what maths are required. The next stage is processing where the student performs the mathematics, and the final stage is encoding, in which the student should record their result appropriately.

White (2009) discussed the developing use of NEA as a diagnostic tool and developed understanding on NEA as a classroom pedagogical strategy. NEA had been applied to determine the type of error made by students in solving the counting problem on algebra and the factors that affect the error made by students algebraic problem (Yusnia & Fitriyani, 2010) and to examine students' errors in solving probability problems using a qualitative approach (Triliana & Asih, 2019). Meanwhile, Anugrah and Kusmayadi (2019) also discussed to define the errors made by students when solving problems involving maximum and minimum derivative values.

Aside from that, NEA was used to ascertain cube and block problem (Farihah & Nashihudin, 2016), geometry problem (Fadhilah & Alfina, 2018) and operation research test (Oktafia et al., 2020). Analysis of student error was expected to help to reflect on solving mathematical problems and became a reference for teachers in choosing strategy, model or learning media to reduce errors made by students (Fitriani et al., 2018). Therefore, the aim of this study is to find out the number of percentage and the level of students’ mistakes using NEA on mathematical communication ability in ratio test.

In solving mathematical problems, it takes a lot more than just a good processing. Students must be able to read and comprehend the issue. They must be able to think critically about what they know about mathematics and select the best solutions for solving a problem. They must use these tactics (processing) and then finally check that they have correctly answered the question. Since there is so much involved in solving problems, it can be difficult to tell whether a student has a problem with their processing or whether they made a mistake somewhere else. Hence, there are some analyses came out to tackle these problems.

Mathematical errors can be factual, procedural, or conceptual, and they can occur for a variety of reasons. For students with lower proficiency, identifying individual faults is very crucial. Besides, Mathematical problem solving is a step-by-step process, with each phase reliant on the success of the previous one for proper execution and, ultimately, a valid response. As a result, by identifying the error, students will be able to determine where the problems occurred and what caused them. After that, it can point students in the direction of possible solutions to their mistakes. Furthermore, it enables examiners to detect specific types and probable patterns of errors, hypothesis why the errors occur, and devise specific interventions to correct the errors discovered.

Methodology

The subject in this study were ninety students of Universiti Teknologi MARA, Permatang Pauh Campus. All the students were in their second year when they undertook Further Calculus for Engineers (MAT455) course. Further Calculus for Engineers is a course for engineering students in degree level at Universiti Teknologi MARA. It involves students from faculty of mechanical engineering, civil engineering, electrical engineering, and also chemical engineering. This three-hour lecture course contains of three main topics which are infinite series, multiple integrals, and vector calculus. The first topic begins with the basic concepts of convergence and the use of various tests to determine the convergence of infinite series. This study focused only on solving convergence of power series using ratio test.

Purposive sampling was used in this study for data collection. The samples taken based on considerations of students' who got the particular question wrong, so the sample was not random. Students' answer script was schemed through and their error were categorise using percentage for each error.

Descriptive method is used to find out the variables and condition of errors in students mathematical solving answers. This study describes student's ability in problem solving in analysis using Newman's Error Analysis (NEA). NEA is according to five errors namely, reading and decoding, comprehension, transformation, process skill, and encoding.

According to the NEA model, the first step, reading and decoding, refers to a student's ability to read a problem and determine the terms or symbols in a query. The second stage examines the students' comprehension of the symbols, expressions, and problems presented in the questions. The third stage is transformation, which refers to students' ability to choose suitable formulae or methods to solve problems, and the fourth stage is exploration of students' process skills in solving problems, including whether the procedure or operation they use is right or incorrect. The final stage is encoding, which examines the students' ability to generate and explain their answers.

Result and Discussion

There are many tests for convergence, however, this study concentrated on convergence of power series using ratio test. Ratio test can be used to find interval of convergence of power series. The following is the definition of ratio test:
Let there be a series $\sum a_n$, the formula for ratio test:

$$r = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$$

where,

If r < 1, the series is convergent.
If r > 1, the series is divergent.
If r = 1, then the series is neither convergent nor divergent (required a different test).

This topic has long solution which consist of six steps overall. Table 1 shows the list of steps in solving convergence of power series by using ratio test.

Table 1
List Of Steps In Solving Convergence of Power Series Using Ratio Test

Solution	Operation
Step 1: Able identify series that need to use ratio test.	Students understand the question and able to write the ratio test formula.
Step 2: Plug-in correct equation into the ratio test formula.	Find a_n and a_{n+1}. Then substitute into the formula correctly.
Step 3: Identify a divide fraction and simplify the fraction.	Change the fraction by multiplying the reciprocal of the fraction.
Step 4: Factor x out of the absolute value.	Factorize function of x correctly before solving the limit.
Step 5: Solve the limit.	Solve the limit based on any technique required.
Step 6: Determine the interval of convergence.	Student able to conclude the interval of convergence for x.

Subsequently, data of all the students' errors were categorise based on NEA's error. Table 2 shows the categorisation of error based on steps of solution for the correct answer in solving convergence of power series. Highest error was on transformation category (38%) and encoding category (38%). Students was poor in solving fraction and limit in ratio test, that directed them to incorrectly conclude the interval of convergence. Non the less, students were good in reading category (5%) and comprehension category (2%).

Table 2
Categorisation of Error Based on Nea and Steps of Solution

Step	NEA's error	Students' error	% of students' error
1	Reading	Student do not understand the question and give a wrong formula.	5 %
2	Comprehension	Give wrong equation of a_n and a_{n+1}. Substitute wrongly in the formula.	2 %
3 & 4	Transformation	Unable to change the fraction by multiplying the reciprocal of the fraction.	38 %
		Factorize function of x incorrectly before solving the limit.	
5	Process skill	Do not solve the limit based on any technique required.	17 %
6	Encoding	Student unable to conclude the interval of convergence.	38 %

Figure 1 show two completed and correct samples using ratio test in solving convergence of power series. It shows all the steps of solution listed in Table 1. Each step is being clearly stated and nicely aligned by the students.

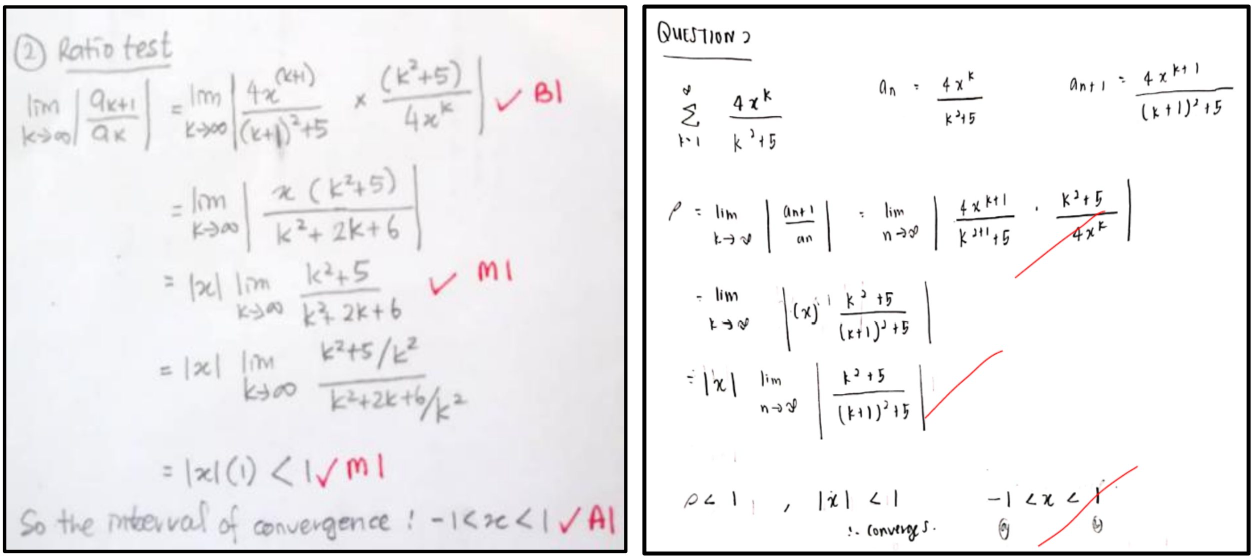

Figure 1. Samples of the Correct Answer

Next, based on student answer scripts, samples of error by the students is presented. Seemed that Step 1 and Step 2 have very small percetange of error made by the students. Therefore, the study focused more on Step 3, Step 4, Step 5, and Step 6. Figure 2 show students' error in solving divide fraction which in step 3. Students had difficulties in solving multiplication of fraction involving equation with many variables. They seemed amiss on the correct way of solving the fraction.

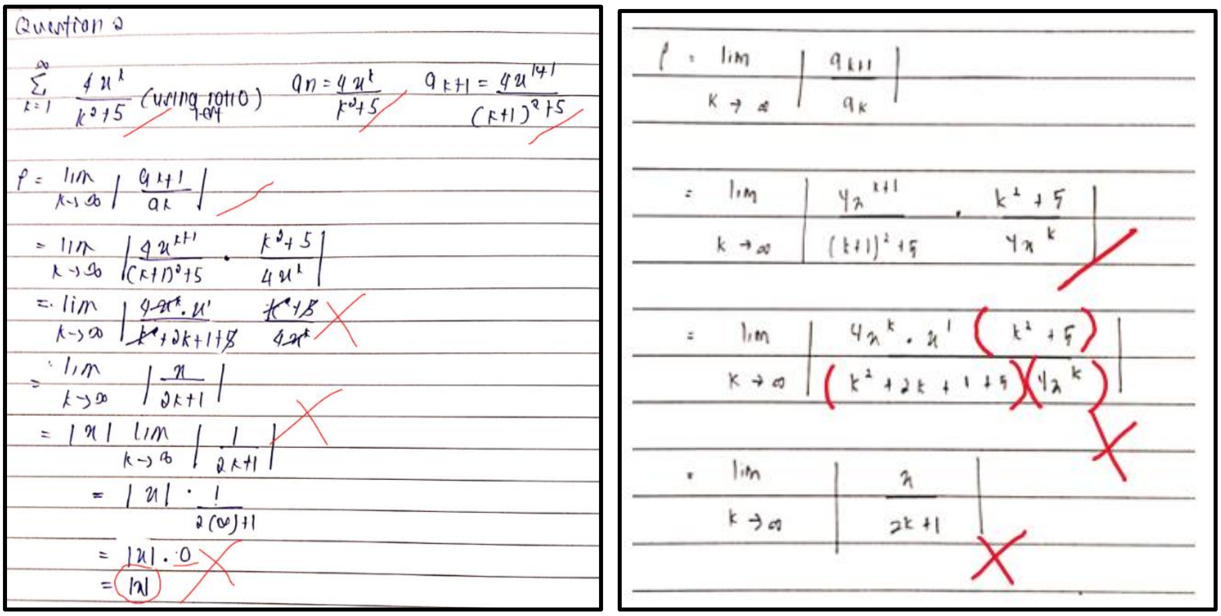

Figure 2. Error in divide fraction in step 3

Figure 3 show students' error in solving factorisation which in step 4. Factorisation of x from the limit would determine correct interval of convergence later in step 6. However, doing error in this step seemed concurrently gave effect on solving the limit.

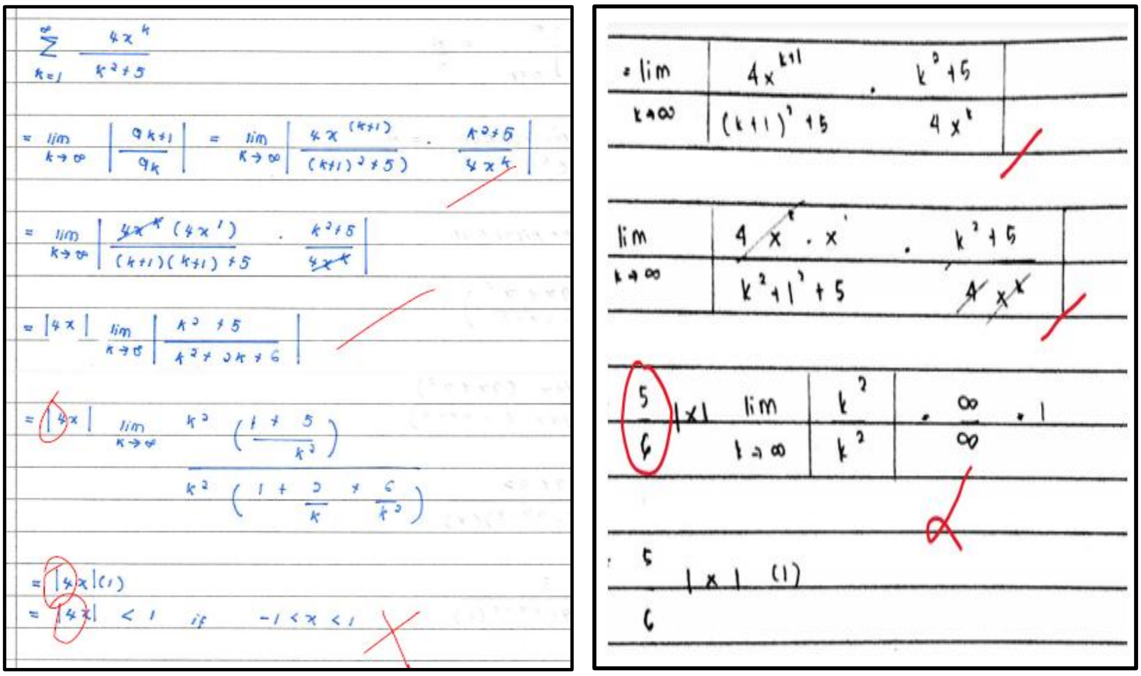

Figure 3. Error in Factorisation in step 4.

As shown in Figure 4, there were error may by the student in solving limit in step 5. Students were unable to solve the limit by using division of highest power of x in the denominator. Even they knew the ratio test, error in solving limit had led them to incorrect answer.

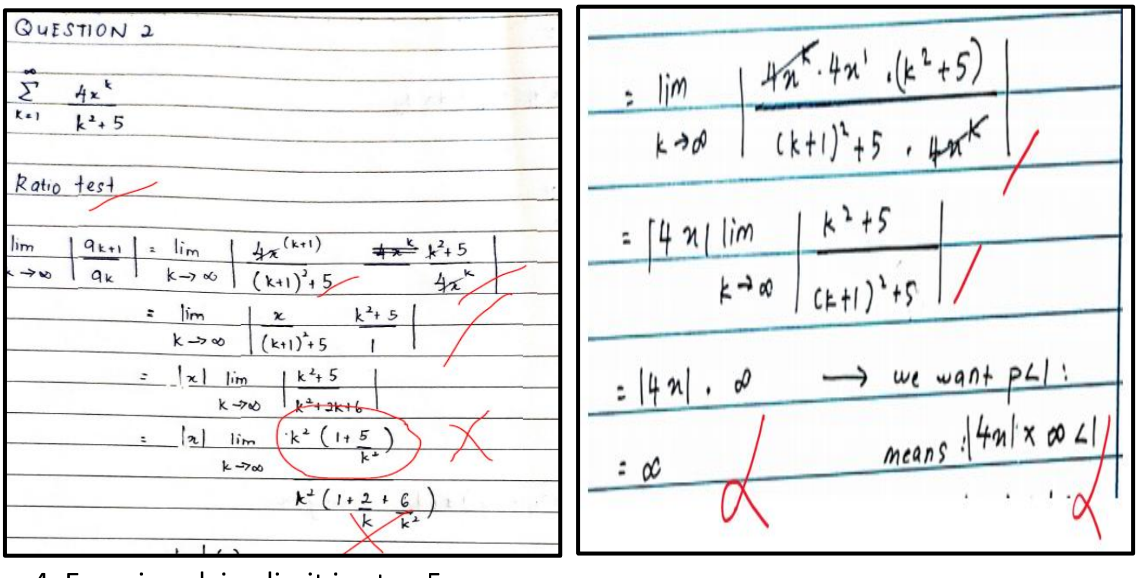

Figure 4. Error in solving limit in step 5.

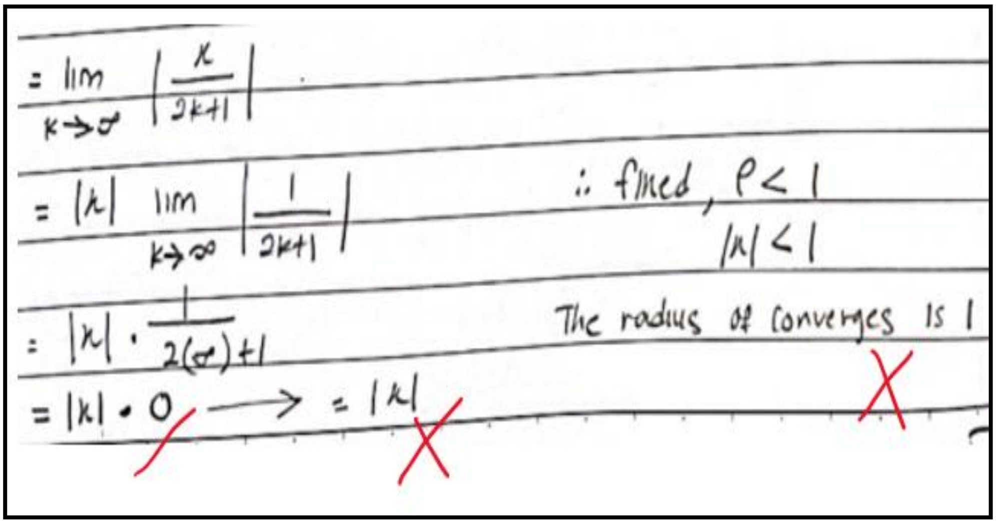

Figure 5. Error in determining interval of convergence in step 6.

Figure 5 shows error in final step where the student concludes their answer and determined the interval of convergence for the power series. Based on the sample, the student was unable to solve the limit correctly, resulting error in interval of convergence.

Conclusion

In order to prevent students doing the same mistakes, they need to be aware of the common mistakes in solving mathematical problems. This study shows that in solving convergence of power series using ratio test, students did five error which are reading error, comprehension error, transformation error, process skill error, and encoding error. Teacher should focus on transformation error and encoding error as these are the common parts that students got wrong. Instead of giving only examples, teacher could give an extra attention on these parts. Students seemed got less error in reading and comprehension parts. These shows that they were able to understand the question and could retrieved the formula. However, they failed to execute the correct solution as they have difficulties in solving fraction and factorisation which led to wrong conclusion in interval of convergence.

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