

# The Welfare Cost of Inflation in Consumer Surplus and Compensating Variation Method: Case Study of Iran

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## Abstract

In this paper using specifying the real money balances, derived the equations of welfare cost of inflation in consumer surplus and compensating variation method, and using annual data over the period 1978 to 2010 estimates the welfare cost of inflation in Iran.

The results indicate that for the semi-log (log-log) specification, estimated using the consumer surplus approach, an increase in the inflation rate from 1 percent to 30 percent increase the welfare cost from 0.038 (0.033) percent of GDP to 3.52 (5.76) percent of GDP. For the log-log specification, estimated using the compensating variation method, an increase in the inflation rate from 1 percent to 30 percent increase the welfare cost from 0.033 to 5.9 percent of GDP

**JEL classification:** C0, E40

**Keywords:** inflation, welfare cost, partial equilibrium, general equilibrium

## 1. Introduction

The effects of inflation on welfare have been the subject of extensive theoretical and empirical analysis. Bailey (1956) and Friedman (1969) compare the welfare cost of inflation to that of an excise tax. Bailey's formula has been derived in a partial-equilibrium analysis. It treats real money balances as a consumption good and inflation as a tax on real balances. The welfare cost is computed by measuring the area under the inverse money demand function.

Since the seminal work of Bailey (1956), economists have devoted considerable effort to measuring the welfare cost of inflation.

Lucas (2000) defined the welfare cost of nominal interest rate to be the percentage income compensation needed to leave the household indifferent between positive and zero nominal interest rate. He employed a general-equilibrium framework and obtained the differential equation of welfare cost. He also employed Bailey's methodology and estimates the welfare cost of inflation based on U.S. time series for 1900–1994 and estimated that the gain from reducing the annual inflation rate from 10 percent to zero is equivalent to an increase in real income of slightly less than one percent.

Serletis and Yavari (2004) use econometrics methods to estimate the welfare cost of inflation for both the US and Canada. They estimate a lower interest elasticity of the demand for money than Lucas uses in his welfare cost calculations and find significantly lower welfare gains from reducing inflation. Serletis and Yavari (2004) use different money demand function to estimate the welfare cost of inflation for a group of Latin American countries. This demand function depends on inflation rate rather than nominal interest rate, because the money demand in developing countries is not very responsive to the central bank's fixed nominal interest rate and the data on the market rate is usually unavailable. Serletis and Yavari (2005) also found the interest rate elasticity in Italy to be -0.26 and claim that reducing interest rates from 14 percent to 3 percent results in a welfare gain of 0.4 percent of income.

Ireland (2009) uses recent advances in econometrics and higher frequency data for the US to perform a similar empirical exercise. He concludes that the welfare gain from reducing inflation in the US is trivial. Cysne (2009) show that, under quasi-linear preferences, Bailey's formula provides an exact measure of the welfare costs of inflation in a Sidrauski general-equilibrium framework.

Yavari and Serletis (2007, 2011) follow Lucas (2000) and estimates the welfare cost of inflation for European and Latin American countries. For the European countries they find that the welfare gain of reducing interest rates from 10 percent to 5 percent ranges from 0.1 to 0.5 percent of GDP, also, they estimates the welfare cost of inflation for 17 Latin American economies. They use annual data, from 1955 to 2000, and recent advances in the field of applied econometrics to estimate the inflation rate elasticity of money demand and report significantly high and differential welfare cost estimates for these economies.

Kimbrough (2012) using quarterly data for the period 1980Q1-1999Q4, measured the welfare cost of inflation in Greece. He concluded that the welfare cost of a 10 percent inflation rate in the range of 0.59 percent to 0.91 percent of GDP.

In this paper, we show that Lucas' measure of welfare cost derived from the Sidrauski general-equilibrium framework to be an upper bound to Bailey's formula in a partial-equilibrium framework. We also use the inflation-based money demand function and advanced econometrics technique to estimate the welfare cost of inflation for Iran using annual data over the period 1978 to 2010.

The organization of the article is as follow. The next section extends the Sidrauski's model (1967a), and follow Lucas (2000) obtains differential equation of welfare cost of inflation in general equilibrium model. Yielding a solution for log-log and semi-log specification, the welfare cost of inflation as a percentage of GDP is obtained and compares its in partial equilibrium model. Section 3 presents empirical evidence regarding the inflation elasticity of money demand, because the money demand in developing countries is not very responsive to the central bank's fixed nominal interest rate, and presents the welfare cost of inflation in general and partial equilibrium model. Section 4 presented summary and conclusion.

## 2. The Model

The economy is populated by infinite lived families, with population growing at rate zero. Sidrauski (1967a) extends the Ramsey's model to allow both consumption  $c$ , and real money balances  $z$ , to enter the utility function. It is assumed that the utility function ( $U$ ) is strictly concave with continuous first and second derivative<sup>1</sup>, strictly increasing in both  $c$  and  $z$ , and that both commodities are not inferior<sup>2</sup>. He further assumes that the total welfare ( $W$ ) associated with any particular time path  $(c_t, z_t)$  can be represented by the utility function. Each household solves the following maximization problem:

$$\text{Max } W = \int_0^{\infty} u(c_t, z_t) e^{-\rho t}, \quad u_c, u_z > 0, \quad u_{cc}, u_{zz} < 0 \quad (1)$$

where  $c$  is real consumption and  $z$  is real money balances. The household can hold its wealth in the form of either money or capital. Its budget constraint is given by

$$c_t = w + rk_t - \dot{k}_t - \dot{z}_t - \pi_t z_t + x \quad (2)$$

where  $k_t$  is the capital asset,  $\pi_t \equiv \frac{\dot{P}_t}{P_t}$  refers to the inflation rate,  $x$  is government transfers,  $w$  and  $r$  are the real wage and the rate of interest and  $P$  is the price level. A dot on the top of a variable represent its rate of change with respect to time. The product  $\pi_t m_t$  is the amount of the inflation tax. Denoting  $a_t \equiv k_t + z_t$ , we can rewrite equation (2) as

$$\dot{a}_t = w_t + r_t k_t - c_t - \pi_t z_t + x \quad (3)$$

This equation gives the rate of change of total wealth. The household chooses consumption  $c_t$  and real money balances  $z_t$  to maximize utility in equation (1) subject to equation (3), taking  $(w_t, r_t, c_t, \pi_t)$  as given. The Hamiltonian function associated with the maximization problem is

$$H = u(c_t, z_t) + \lambda_t [w_t + r_t k_t - c_t - \pi_t z_t + x] + v_t (a_t - k_t - z_t)$$

Let  $\lambda_t$  be the co-state variable and  $v_t$  is a multiplier. The first order conditions with respect to  $c$ ,  $z$ ,  $k$  and  $a$ , respectively, are

$$u_c(c_t, z_t) = \lambda_t \quad (4)$$

$$u_z(c_t, z_t) = \lambda_t \pi_t + v_t \quad (5)$$

$$\lambda_t r_t = v_t \quad (6)$$

$$v_t = \rho \lambda_t - \dot{\lambda}_t \quad (7)$$

The transversality condition ruling out the Ponzi game is given by

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t a_t = 0 \quad (8)$$

In the steady state  $\dot{z} = \dot{\lambda} = \dot{a} = 0$  and the first conditions imply that

$$\frac{u_z(c, z)}{u_c(c, z)} = \pi + r \quad (9)$$

So that the marginal rate of substitution between consumption and real money balances is equal to the nominal interest rate  $i$ , which has therefore interpretation of price of money services.

As in Lucas (2000), we define the welfare cost of positive nominal rate of interest ( $w(i)$ ) to be the percentage income compensation needed to leave the household indifferent between  $i$  and 0. That is,  $w(i)$  is defined as the solution to

$$U[(1 + w(i))y, m(i)y] = U[y, m(0)y] \quad (10)$$

<sup>1</sup> -This condition implies that  $u_{cc} < 0, u_{zz} < 0$  and  $J = u_{cc}u_{zz} - u_{zc}^2 > 0$ .

<sup>2</sup> -This requires:  $J_1 = u_{zz} - u_{cz}z/u_c < 0$  and  $J_2 = u_{cc}u_z/u_c - u_{cz} < 0$ .

Where  $m = \frac{z}{y}$  is the money-income ratio and  $i$  is the nominal rate of interest. Let  $\phi: R^+ \rightarrow R$  be a strictly increasing and concave function. Lucas (2000) assuming a homothetic current period utility function  $u$  is given by

$$u(c, z) = \frac{1}{1-\sigma} [c\phi(\frac{z}{c})]^{1-\sigma}, 0 < \sigma, \sigma \neq 1 \tag{11}$$

With assumed this functional form, definition in (10) is equivalent to

$$(1 + w(i))\phi\left(\frac{m(i)}{1+w(i)}\right) = \phi(m(0)) \tag{12}$$

Let  $m(i)$  be given and  $\psi(m)$  be the inverse function. If  $c/y = 1$  (which will hold along any equilibrium path, then (9) implies that the function  $\phi$  satisfies the differential equation

$$\frac{\phi'(m)}{\phi(m) - m\phi'(m)} = i \tag{13}$$

Equation (13) implies that the function  $\phi$  satisfies this differential equation

$$\phi'(m) = \frac{\psi(m)}{1+m\psi(m)} \phi(m) \tag{14}$$

Lucas (2000) with differentiating (12) through with respect to  $i$ , and use (13) obtains differential equation

$$w'(i) = -\psi\left(\frac{m(i)}{1+w(i)}\right) m'(i), \quad w(0) = 0 \tag{15}$$

For any given money demand function, equation (15) is readily solved numerically for an exact welfare cost function. Lucas (2000) contrasts between two competing specifications for money demand. One, inspired by Meltzer (1963), relates the natural logarithm of  $m$ , a ratio of money balances to nominal income, and the natural logarithm of a short-term nominal interest rate. In this paper, the welfare cost is computed by measuring the area under the inverse money demand function and demand function depends on inflation rate rather than nominal interest rate, because the money demand in developing countries is not very responsive to the central bank's fixed nominal interest rate and the data on the market rate is usually unavailable. For the log-log demand function

$$m(\pi) = A\pi^{-\eta} \tag{16}$$

where  $A > 0$  is a constant and  $\eta > 0$  measures the absolute value of the inflation elasticity of money demand function. Another specification, adapted from Cagan (1956), links the log of  $m$  to the level of  $i$  via the following equation:

$$m(\pi) = B e^{-\xi\pi} \tag{17}$$

where  $B > 0$  is a constant and  $\xi > 0$  measures the absolute value of the semi-elasticity of money demand function with respect to the inflation rate. Equation (15) implies

$$w'(\pi) = \eta A \pi^{-\eta} (1 + w(\pi))^{-\frac{1}{\eta}} \tag{18}$$

Yielding a solution for log-log specification, the welfare cost of inflation as a percentage of GDP is obtained as follows:

$$w(\pi) = -1 + (1 - A\pi^{1-\eta})^{-\frac{\eta}{1-\eta}} \tag{19}$$

By applying the methods outlined in Bailey (1956), Lucas (2000) transformed the evidence on money demand in to the welfare cost estimate. Note Bailey (1956) described the welfare cost of inflation as the area under the inverse money demand function, or the consumers's surplus, that could be gained by reducing the interest rate to zero from an existing (average or steady-state) value. So if  $m(i)$  is the estimated function, and  $\psi(m)$  is the inverse function, then the welfare cost can be defined as:

$$w(i) = \int_{m(i)}^{m(0)} \psi(x) dx = \int_0^i m(x) dx - im(i) \tag{20}$$

As seen from Equation (20), obtaining a measure for the welfare cost amounts to, integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue  $im(i)$  to deduce the deadweight loss. When the money demand function is given by equation (16), the welfare cost of inflation as a percentage of GDP is obtained as follows:

$$w(\pi) = \left[ \frac{A}{1-\eta} \pi^{1-\eta} \right]_0^\pi - \pi A \pi^{-\eta} = A \frac{\eta}{1-\eta} \pi^{1-\eta} \tag{21}$$

For a semi-log money demand specification, as equation (17),  $w(\pi)$  is obtained by the following formula:

$$w(\pi) = \left[ -\frac{B}{\xi} e^{-\xi\pi} \right]_0^\pi - \pi B e^{-\xi\pi} = \frac{B}{\xi} [1 - (1 + \xi\pi)e^{-\xi\pi}] \tag{22}$$

As can be seen from equations (19), (21) and (22), an estimate of the inflation elasticity of money demand is crucial in evaluating the welfare cost of inflation, and, hence, we first need to obtain the long-run relationship between the ratio of money balance to income and inflation rate. We use the estimates of  $\eta$  and  $\xi$  obtained from the long-horizon regression, discussed in Section 3.

### 3. Welfare cost estimates

To investigate the welfare cost of inflation, we use annual data from 1978 to 2010 for Iran, which, in turn, are obtained from the Central Bank of Islamic Republic of Iran and the Data Base of the World Bank (WDI). The variables used in this study are the money balances ratio,  $rm2$ , generated by dividing the total liquidity by the nominal income (nominal GDP), and inflation rate. Further, for the estimation of the log-log specification both the ratio of money balances and the inflation rate are transformed into their logarithmic values, and are denoted by  $lrm2$  and  $linf$ , respectively. We use the semi-log and log-log money demand function and the econometric methodology to get an estimate of the inflation rate elasticity,  $\xi$  and  $\eta$  respectively. The welfare cost is computed by measuring the area under the inverse money demand function and demand function depends on inflation rate rather than nominal interest rate, because the money demand in developing countries is not very responsive to the central bank's fixed nominal interest rate and the data on the market rate is usually unavailable

In time series analysis, we first test for stochastic trends (unit roots) in the autoregressive representation of the  $lrm2$ ,  $linf$  and  $inf$  series. In this regard, we performed tests of stationary on our variables using the Augmented–Dickey–Fuller (ADF) unit root test and find that  $lrm2$  and  $linf$  are  $I(0)$  and  $inf$  is  $I(1)$ . The ADF test results are presented in Table 1.

Table1. ADF test with constant only and with constant and trend (Lag 4), 1978-2010

ADF test with constant			ADF test with constant and trend	
Variables	Levels	First difference	Levels	First difference
<i>lrm2</i>	-3.11**	-5.73**	-3.81**	-6.12**
<i>linf</i>	-3.11**	-5.30**	-3.11**	-5.19**
<i>inf</i>	-2.89	-5.38**	-3.40	-5.38**

Notes: \*\* indicate significant at the 5% level. Critical values with no trend and trend at the 5% significant level are -2.97 and -3.57, respectively.

Source: Researchers computations

We also use Auto Regressive Distributed Lag (ARDL) approach to estimate long run relationship between the variables of the models. This method is implemented regardless of whether the underlying variables are I(0) or I(1), or fractionally integrated. The ARDL procedure is represented by the following equation:

$$\phi(L, P)Y_t = \sum_{i=1}^k \alpha_i(L, q_{it}) X_{it} + \delta'w_t + u_t \tag{23}$$

Where

$$\phi(L, p) = 1 - \phi_1L - \phi_2L^2 - \dots - \phi_pL^p, \text{ and}$$

$$\alpha_i(L, q_i) = \alpha_{i0} + \alpha_{i1}L + \alpha_{iq}L^q, i = 1, 2, \dots, k$$

Where  $Y_t$  denotes the dependent variable,  $X_{it}$  is the  $i^{\text{th}}$  independent variables,  $L$  is a lag operator and  $w$  represents the deterministic variables employed, including intercept terms, dummy variables, time trends and other exogenous variables. The optimum lag length is generally determined by either the Akaike Information Criterion (AIC) or the Schwarz Bayesian Criteria (SBC). The long run relationship between variables that specify the model can be tested as following:

$$H_0 : \sum_{i=1}^p \phi_i - 1 \geq 0, \quad H_1 : \sum_{i=1}^p \phi_i - 1 < 0 \tag{24}$$

Where  $H_0$  represents that the long run relationship is not exist and can be tested with Bannerjee, Dolado and Mestre (1998) approach as following manner:

$$t = \frac{\sum_{i=1}^p \hat{\phi}_i - 1}{se_{\hat{\phi}_i}} \tag{25}$$

Where  $\hat{\phi}_i$  is coefficient of dependent variable. When calculated  $t$  test is greater than the critical value, which is calculated by Bannerjee, Dolado, and Mestre (1998),  $H_0$  rejected and there are the long run relations between the variables in both log-log and semi-log money demand functions. The results of co-integration test are presented in Table 2.

Tabel 2. The results of co-integration test

Log-log forms			Semi-log forms	
T	t-test	critical value	t-test	critical value
25	-3.576	-3.35**	-3.133	-2.95***
50	-3.575	-3.28**	-3.133	-2.93***

Notes: \*\* and \*\*\* indicate significant at the 5% and 10% significant level, respectively.

Source: Researchers computations

The results in table 2. Indicate that in both Log-log and Semi-log forms there are long run relation between inflation and money balances ratio.

We first estimate inverse money demand functionin with both log-log and semi-log forms in equations (16) and (17), respectively. For the period 1978-2010 estimated long run relations equations are

$$Ln\pi_t = -0.33 + 0.84Lrm2 \tag{26}$$

(1.81) (0.481)

$$\pi_t = -27.80 + 12.34Lrm2 \tag{27}$$

(40.15) (10.73)

The numbers in parentheses indicate standard deviation. With our estimates values of  $\alpha_1 = -0.33$ , and  $\beta_1 = 0.84$  in equation (26) and  $\alpha_2 = -27.80$ , and  $\beta_2 = 12.34$  in equation (27), we can calculate A and  $\eta$  in equation (16) and B and  $\xi$  in equation (17)<sup>3</sup>. So, our calculations show that  $\eta = 1.19$ ,  $A = 1.481$ ,  $\xi = .081$ , and  $B = 9.505$  in equations (16) and (17), respectively.

We are now in a position to obtain the welfare cost estimates of inflation, using both Bailey’s (1956) consumer surplus approach and Lucas’ (2000) compensating variation method. The welfare cost estimates, based on the values of  $\eta$ ,  $\xi$ , A and B, given by the equation (21) for log-log and equation (22) for semi-log specification in consumer surplus method and equation (20) for log-log specification in compensating variation method. For the semi-log (log-log) specification, estimated using the consumer surplus approach, an increase in the inflation rate from 1 percent to 30 percent increase the welfare cost from 0.038 (0.033) percent of GDP to 3.52 (5.76) percent of GDP. For the log-log specification, estimated using the compensating variation method, an increase in the inflation rate from 1 percent to 30 percent increase the welfare cost from 0.033 to 5.9 percent of GDP. These results indicate in figure.1, 2 and 3.

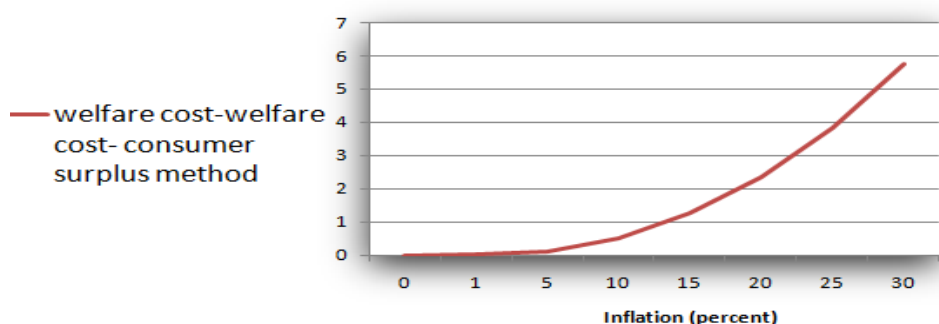


Fig. 1. Welfare cost of inflation in consumer surplus method-(log log)

Source: Researchers result

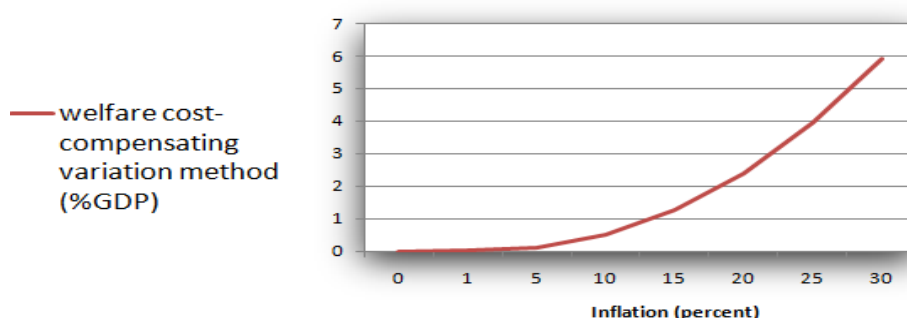


Fig. 2. Welfare cost of inflation in compensating variation method-(log log)

Source: Researchers result

<sup>3</sup> -  $\eta = \frac{1}{\beta_1}$ ,  $A = \exp(\alpha_1)^{-\eta}$ ,  $\xi = \frac{1}{\beta_2}$  and  $B = \exp(\alpha_2)^{-\xi}$

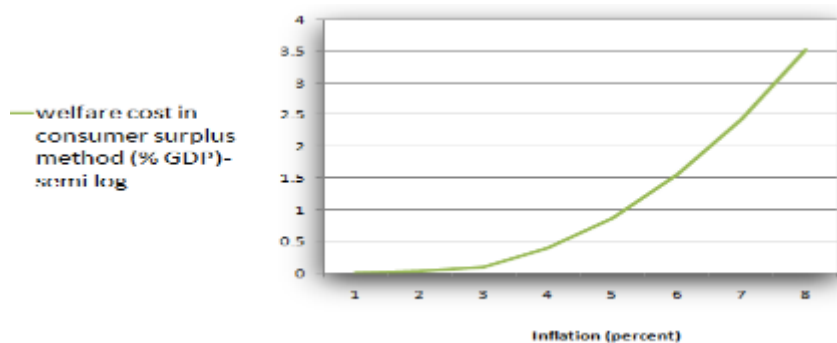


Fig. 3. Welfare cost of inflation in consumer surplus method-(Semi log)

**Source:** Researchers result

In Fig.1, 2 and 3, we plot the welfare cost function,  $w(\pi)$ , based on Eqs. (19), (21) and (22) for Iran. Clearly, the welfare cost estimates in Lucas' (2000) compensating variation method based on Eqs. (19) to be an upper bonded on Bailey's (1956) consumer surplus approach based on Eqs. (21). The welfare cost functions in Figure.1, 2 and 3 are convex, indicating the increasing marginal welfare cost of inflation.

#### 4. Conclusion

Lucas (2000) has shown that Bailey's formula for the welfare costs of inflation can be regarded as a very good approximation to general-equilibrium measures originating from the Sidrauski models. Deepening such a result in this paper, we show that welfare cost for the log-log specification, estimated using the compensating variation method in general-equilibrium, to be an upper bound to consumer surplus approach in Bailey's partial-equilibrium. We use the inflation-based money demand function and advanced econometrics technique to estimate the welfare cost of inflation for Iran using annual data over the period 1978 to 2010.

The results indicate that for the semi-log (log-log) specification, estimated using the consumer surplus approach, an increase in the inflation rate from 1 percent to 30 percent increase the welfare cost from 0.038 (0.033) percent of GDP to 3.52 (5.76) percent of GDP. For the log-log specification, estimated using the compensating variation method, an increase in the inflation rate from 1 percent to 30 percent increase the welfare cost from 0.033 to 5.9 percent of GDP

#### References

- Baily, M. (1956). The Welfare Cost of Inflationary Finance, *Journal of Political Economy*, 64: 93-110.
- Friedman, M. (1969). *The Optimum Quantity of Money and Other Essays*, Chicago: Aldine.
- Ireland, Peter (2009). On the Welfare Cost of Inflation and the Recent Behavior of Money Demand, *American Economic Review* 99: 1040–1052.
- Kimbrough, K. (2012). The Welfare Cost of Inflation in Greece, *South-Eastern Europe Journal of Economics* 1, 41-52
- Lucas Jr., R.E. (2000). Inflation and Welfare, *Econometrica* 68: 247–274.
- Meltzer, E. (1963a). The Demand for Money: The Evidence from Time Series, *Journal of Political Economy* 71: 219-246.

- Sidruski, M. (1967a). Rational Choice and Patterns of Growth in a Monetary Economy, *American Economic Review* 57: 534-544.
- Serletis A, K, Yavari. (2004). The Welfare Cost of Inflation in Canada and the United States, *Econ Lett* 84:199–204.
- Serletis, A., and K, Yavari. (2005). The Welfare Cost of Inflation in Italy, *Applied Economics Letters* 12: 165-168.
- Serletis, A and K,Yavari. (2007). On the Welfare Cost of Inflation in Europe, *Applied Economic Letters* 14: 111–113.
- Yavari, K and A, Serletis. (2011). Inflation and Welfare in Latin America, *Open Economies Review* 22: 39–52.
- Uwilingiye, J and R. Gupta. ( 2008). Measuring the Welfare Cost of Inflation in South Africa, *South African Journal of Economics*, 76(1): 16-25.