Forecasting of Exchange Rate Volatility between Naira and US Dollar Using GARCH Models

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Abstract
Exchange rates are important financial problem that is receiving attention globally. This study investigated the volatility modeling of daily Dollar/Naira exchange rate using GARCH, GJR-GARCH, TGRACH and TS-GARCH models by using daily data over the period June 2000 to July 2011. The aim of the study is to determine volatility modeling of daily exchange rate between US (Dollar) and Nigeria (Naira). The results show that the GJR-GARCH and TGARCH models show the existence of statistically significant asymmetry effect. The forecasting ability is subsequently assessed using the symmetric lost functions which are the Mean Absolute Error (MAE), Root Mean Absolute Error (RMAE), Mean Absolute Percentage Error (MAPE) and Theil inequality Coefficient. The results show that TGARCH model provide the most accurate forecasts. This model will captured all the necessary stylize facts (common features) of financial data, such as persistent, volatility clustering and asymmetric effects.
Key words: Volatility, GARCH, Asymmetric models, Exchange Rates

JEL: D53, F31, F43

1.0 Introduction
Most research have been made on forecasting of financial and economic variables through the help of researchers in the last decades using series of fundamental and technical approaches yielding different results. The theory of forecasting exchange rate has been in existence for many centuries where different models yield different forecasting results either in the sample or out of sample. Exchange rate which means the exchange one currency for another price for which the currency of a country (Nigeria) can be exchanged for another country's currency say (dollar). A correct exchange rate do have important factors for the economic growth for most developed countries whereas a high volatility has been a major problem to economic of series of African countries like Nigeria. There are some factors which definitely affect or influences exchange rate like interest rate, inflation rate, trade balance,

general state of economy, money supply and other similar macro – economic giants' variables. Many researchers have used multi-variate regression approach to study and to predict the exchange rate base on some of these listed variables, but this has a limitation in the sense that macro- economic variables are available at most monthly period and precisely modeling of such explanatory variable on exchange rate do make explains that a change in unit of each macro- economic variables will definitely lead to a proportion change in the exchange rate. In this view why not exchange rate explains itself that is with the little information of its self can predict its current value and its future value through the use of robust time series or technical model or approaches. (Onasanye *et al,* 2013).

The uncertainty of the exchange rate shows how much economic behaviors are not able to perceive the directionality of the actual or future volatility of exchange rate, that is, it is a different concept from the volatility of the exchange rate itself in that it means that the more forecast errors of economic behaviors made, the higher the trends in the uncertainty of the exchange rate are shown (Yoon and Lee, 2008). The volatility of financial assets has been of growing area of research (see Longmore and Robinson (2004) among others). The traditional measure of volatility as represented by variance or standard deviation is unconditional and does not recognize that there are interesting patterns in asset volatility; e.g., time-varying and clustering properties. Researchers have introduced various models to explain and predict these patterns in volatility. Engle (1982) introduced the autoregressive conditional heteroskedasticity (ARCH) to model volatility. Engle (1982) modeled the heteroskedasticity by relating the conditional variance of the disturbance term to the linear combination of the squared disturbances in the recent past. Bollerslev (1986) generalized the ARCH model by modeling the conditional variance to depend on its lagged values as well as squared lagged values of disturbance, which is called generalized autoregressive conditiona l heteroskedasticity (GARCH). Since the work of Engle (1982) and Bollerslev (1986), various variants of GARCH model have been developed to model volatility. Some of the models include IGARCH originally proposed by Engle and Bollerslev (1986), GARCH in- Mean (GARCH-M) model introduced by Engle, Lilien and Robins (1987),the standard deviation GARCH model introduced by Taylor (1986) and Schwert (1989), the EGARCH or Exponential GARCH model proposed by Nelson (1991), TARCH or Threshold ARCH and Threshold GARCH were introduced independently by Zakoïan (1994) and Glosten, Jaganathan, and Runkle (1993), the Power ARCH model generalised by Ding, Zhuanxin, C. W. J. Granger, and R. F. Engle (1993) among others. The modeling and forecasting of exchange rates and their volatility has important implications for many issues in economics and finance. Various family of GARCH models have been applied in the modeling of the volatility of exchange rates in various countries. Taylor (1987) and more recently West and Chow (1995) examined the forecast ability of exchange rate volatility using a number of models including ARCH using five U.S. bilateral exchange rate series. They found that generalised ARCH (GARCH) models were preferable at a one week horizon, whilst for less frequent data, no clear victor was evident. Some other studies on the volatility of exchange rates include Meese and Rose 379 (1991), McKenzie (1997), Christian (1998), Longmore and Wayne Robinson (2004), Yang (2006) Yoon and Lee (2008) among others. Little or no work has been done on modeling exchange rate volatility in Nigeria particularly using GARCH models. The exchange rate volatility has implications for many issues in the arena of finance and economics. Such issues include impact of foreign exchange rate volatility on derivative pricing, global trade patterns, countries balance of payments position, government policy making decisions and international capital budgeting.

1.1 Effect of Structural Adjustment Program

In 1986, Nigeria adopted the structural adjustment programme (SAP) of the IMF/World Bank. With the adoption of SAP in 1986, there was a radical shift from inward-oriented trade policies to out ward –oriented trade policies in Nigeria. These are policy measures that emphasize production and trade along the lines dictated by a country's comparative advantage such as export promotion and export diversification, reduction or elimination of import tariffs, and the adoption of market-determined exchange rates. Some of the aims of the structural adjustment programme adopted in 1986 were diversification of the structure of exports, diversification of the structure of production, reduction in the over-dependence on imports, and reduction in the over-dependence on petroleum exports. The major policy measures of the SAP were:

· Deregulation of the exchange rate
· Trade liberalization
· Deregulation of the financial sector
· Adoption of appropriate pricing policies especially for petroleum products.
· Rationalization and privatization of public sector enterprises and
· Abolition of commodity (Onasanye *et al,* 2013).

2.   Materials and Methods

Data:

The daily exchange rates of the US Dollar against the Euro for the period 4th January, 1999 to 26th April, 2013 are used. These make a total of 3602 observations of the spot price and are converted for the needs of fitting the model to a logarithmic returns series. If the price series is denoted $\{e_t\}$, then the log returns series r_t is such that,

$$r_t = \log\left(\frac{e_t}{e_{t-1}}\right) \qquad\qquad 1$$

Where e_t is the Euro-dollar exchange rate at time *t* and e_{t-1} represent Euro-dollar exchange rate at time *t-1*. The r_t of equation (1) will be used in observing the volatility of the exchange rate between euro and United State Dollar over the period 1999-2013.

2.1   GARCH (1, 1) Model:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \qquad\qquad 2$$

Where:

β_1 Measures the extent to which a volatility shock today feed through into the next period's volatility. $(\alpha_1 + \beta_1)$ Measures the rate at which this effect lies over time. h_{t-1} is the volatility at day *t-1.*

2.2   GARCH Model Extensions:

In most cases, the basic GARCH model provides a reasonably good model for analyzing financial time series and estimating conditional volatility. However, there are some aspects of the model which can be improved so that it can better capture characteristics and dynamics of a particular time series. Since bad news (negative shocks) tends to have a large impact on volatility than good news (positive shocks), hence there is need to talk about the GARCH

extensions model and we restricted our analysis to the more popular models of asymmetric volatility, such as EGARCH, TGARCH, IGARCH, TS-GARCH, APARCH, GJR-GARCH, etc.

2.2.1 EGARCH Model:

[9] proposed the exponential GARCH (EGARCH) model to allow for leverage effects. The model has the following representation:

$$\log h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \frac{|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i}}{h_{t-i}^{\frac{1}{2}}} + \sum_{j=1}^{q} b_j \log h_{t-j} \qquad\qquad 3$$

Where, γ_i = leverage effect coefficient. (if γ_i >0 it indicates the presence of leverage effect). Note that when ε_{t-i} is positive or there is "good news", the total effect of ε_{t-i} is $(1 + \gamma_i)|\varepsilon_{t-i}|$; in contrast, when ε_{t-i} is negative or there is "bad news" the total effect of ε_{t-i} is $(1 - \gamma_i)|\varepsilon_{t-i}|$. Bad news can have a large impact on volatility, and the value of γ_i would be expected to be positive.

2.2.2 TGARCH Model:

Another GARCH variant that is capable of modeling leverage effects is the Threshold GARCH (TGARCH) model, which has the following form:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \gamma_i s_{t-i} \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \qquad\qquad 4$$

Where:

$$S_{t-i} = \begin{cases} 1 \, if \; \varepsilon_{t-i} < 0 \\ 0 \, if \; \varepsilon_{t-i} \geq 0 \end{cases}$$

γ_i = leverage effects coefficient. (if γ_i >0 it indicates the presence of leverage effect). That is depending on whether ε_{t-i} is above or below the threshold value of zero, ε_{t-i}^2 has different effects on conditional variance h_t : when ε_{t-i} is positive, the total effects are given by $\alpha_i \varepsilon_{t-i}^2$ and when ε_{t-i} is negative, the total effects are given by $(\alpha_i + \gamma_i)\varepsilon_{t-i}^2$. So one would expects γ_i to be positive for bad news to have larger impacts. This model also known as the GJR model ([5]) proposed essentially the same model.

2.2.3 GJR-GARCH Model:

The GJR-GARCH model is another volatility model that allows asymmetric effects. This was introduced by [6]. The general specification of this model is of the form:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \left(\alpha_i \varepsilon_{t-1}^2 + \gamma_i s_{t-i}^- \varepsilon_{t-1}^2 \right) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \qquad\qquad 5$$

Where: s_{t-i}^- is a dummy variable which takes the value of 1 when γ_i is negative and 0 when γ_i is positive. In this GJR-GARCH model, it is supposed that the impact of ε_t^2 on the conditional variance σ_t^2 differs when ε_t^2 is positive or negative. A nice aspect of the GJR-GARCH model is that it is easy to test the null hypothesis of no leverage effects. In fact, γ_1 =….= γ_q =0 means that the news impact curve is symmetric, i.e. past negative shocks have the same impact on today's volatility as positive shocks.

2.2.4 Asymmetric Power ARCH (APARCH) Model:

Asymmetric Power ARCH (APARCH) model, introduced by [3].This model is able to accommodate asymmetric effects and power transformations of the variance. Its specification for the conditional variance is the following:

$$\sigma_t^{\delta} = \omega' z_t + \sum_{i=1}^{q} \alpha_i \left(\left| u_{t-i} \right| - \gamma_i u_{t-i} \right) + \sum_{j-1}^{p} \beta_j \sigma_{t-j}^{\delta} \qquad 6$$

Where:

$\sigma_t \equiv \sqrt{ht}$, the parameter δ (assumed positive but typically ranging between 1 and 2) performs a Box-Cox transformation and γ captures the asymmetric effects.

2.2.5   TS-GARCH MODEL
The TS-GARCH model developed by Taylor (1986) and Schwert (1990) is another popular model used to capture the information content in the thick tails, which is common in the return distribution of speculative prices. The specification of this model is based on standard deviations and is as follows:

$$(h_t^{1/2}) = w + \sum_{i=1}^{p} (\phi \left| \varepsilon_{t-i} \right|) + \sum_{j=1}^{q} \beta_j (h_{t-j}^{1/2}) \qquad 7$$

2.3   Calculating the *optimal h-step ahead* forecast
It important to note that to calculate the *optimal h-step ahead* forecast of y_t , the forecast function obtained by taking the conditional expectation of y_{T+h} (where T is the sample size) is used. So, for example, in the case of the AR (1) model:

$$y_t = \delta + \rho y_{t-1} + \varepsilon_t \qquad 7$$

where $\varepsilon \sqcup IID(0, \sigma^2)$, the optimal *h*-step ahead forecast is:

$$E\left(y_{T+h} \left| \Omega_T \right. \right) = \delta + \rho y_{T+h-1} \qquad 8$$

Where Ω_T is the relevant information set. Therefore the optimal one-step ahead forecast of y_t is simply $\delta + \rho y_{t-1}$. While the forecasting function for the conditional variances of ARCH and GARCH models are less well documented than the forecast function for conventional ARIMA models (see [6], chapter 5, for detailed information on the latter), the methodology used to obtain the optimal forecast of the condition variance of a time series from a GARCH model is the same as that used to obtain the optimal forecast of the conditional mean. Further details on the forecast function for the conditional variance of a GARCH (p,q) process is given below:
Consider the equation for the conditional variance in a GARCH (p,q) model:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \qquad 9$$

Taking conditional expectations and assuming a sample size of *T* and for convenience that the parameters in the forecast functions are known, the forecast function for the optimal h-step ahead forecast of the conditional variance can be written:

$$E\left(h_{T+h|\Omega_T} \right) = \alpha_0 + \sum_{i=1}^{q} \alpha_i E\left(u_{T+h-i}^2 \left| \Omega_T \right. \right) + \sum_{j=1}^{p} \beta_j E\left(h_{T+h-j|\Omega_T} \right) \qquad 10$$

Where Ω_T is the relevant information set. $E\left(u_{T+i}^2 \middle| \Omega_T\right) = E\left(h_{T+i} \middle| \Omega_T\right)$ for $i > 0$,

$E\left(u_{T+i}^2 \middle| \Omega_T\right) = u_{T+i}^2$ for $i \le 0$ and $E\left(h_{T+i} \middle| \Omega_T\right) = h_{T+i}$ for $i > 1$. $E\left(h_{T+i} \middle| \Omega_T\right)$ is computed recursively. Thus the one-step ahead forecast of h_T is given by:

$$E\left(h_{T+i} \middle| \Omega_T\right) = \alpha_0 + \alpha_1 u_T^2 + \beta_1 h_T \qquad\qquad 11$$

The forecast of the conditional variance for GARCH-M models can be obtained in a similar way. Clearly, the forecast functions for some of the extensions of the original GARCH specifications will more difficult to drive. For example, in the GJR-GARCH model recall that the conditional variance is given by:

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma_1 u_{t-1}^2 I_{t-1} + \beta_1 h_{t-1} \qquad\qquad 12$$

Where $I_{t-1} = 1$ if $u_{t-1} > 0$ and $I_{t-1} = 0$ otherwise. Unless $\gamma_1 = 0$, forecasts of the indicator function I_{t-1} need to be computed. The sign of u_{t-1} and therefore the forecasts of I_{t-1} will depend on the assumed distribution for ε_t.

2.4 Volatility Forecasts Comparison:

Volatility forecasts comparison was conducted for one-step ahead horizon in terms of Root Mean Square Error (RMSE), Mean Square Error (MSE), Mean Absolute Error (MAE), Mean Percentage Error (MPE), Mean Error (ME), Mean Logarithm of Absolute Error (MLAE) and Theil Inequality Coefficient.

In order to estimate the forecasting performance of some models or to compare several models, we should define error functions. The following are the most used error functions:

1. Root Mean Square Error (RMSE) = $\sqrt{\dfrac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t - \sigma_t\right)^2}$ 13

2. Mean Absolute Error (MAE) = $\dfrac{1}{N}\sum_{t=1}^{N}\left|\hat{\sigma}_t - \sigma_t\right|$ 14

3. Mean Absolute Percent Error (MAPE) = $\dfrac{1}{N}\sum_{t=1}^{N}\dfrac{\left|\hat{\sigma}_t - \sigma_t\right|}{\sigma_t}$ 15

4. Theil Inequality Coefficient = $\dfrac{\sqrt{\dfrac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t - \sigma_t\right)^2}}{\sqrt{\dfrac{1}{N}\sum_{t=1}^{N}\hat{\sigma}_t^2} + \sqrt{\dfrac{1}{N}\sum_{t=1}^{N}\sigma_t^2}}$ 16

Where N is the number of out-of-sample observations, σ_t is the actual volatility at forecasting period t measured as the square daily return, and $\hat{\sigma}_t$ is the forecast volatility at t.

Note that the first three forecast error statistics depend on the scale of the dependent variable. These should be used as relative measures to compare forecasts for the same series across different models; the smaller the error, the better the forecasting ability of that model according to that criterion. The Theil inequality coefficient always lies between zero and one, where zero indicates a perfect fit.

Note also that the mean squared forecast error can be decomposed as:

$$\frac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t - \sigma_t\right)^2 = \left(\left(\frac{1}{N}\sum_{t=1}^{N}\hat{\sigma}_t\right) - \bar{y}\right)^2 + \left(\delta_{\hat{y}} - \delta_y\right)^2 + 2\left(1-r\right)\delta_{\hat{y}}\delta_y$$

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Where $\frac{1}{N}\sum_{t=1}^{N}\hat{\sigma}_t, \bar{y}, \delta_{\hat{y}}, \delta_y$ are the means and (biased) standard deviations of $\hat{\sigma}_t$ and σ_t, and r is the correlation between $\hat{\sigma}$ and σ. The proportions are defined as:

Bias proportion = $\dfrac{\left(\dfrac{1}{N}\sum_{t=1}^{N}\hat{\sigma}_t - \bar{y}\right)^2}{\dfrac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t - \sigma_t\right)^2}$

18

Variance proportion = $\dfrac{\left(\delta_{\hat{y}} - \delta_y\right)^2}{\dfrac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t - \sigma_t\right)^2}$

19

Covariance proportion = $\dfrac{2\left(1-r\right)\delta_{\hat{y}}\delta_y}{\dfrac{1}{N}\sum_{t=1}^{N}\left(\hat{\sigma}_t - \sigma_t\right)^2}$

20

• The bias proportion tells us how far the mean of the forecast is from the mean of the actual series.
• The variance proportion tells us how far the variation of the forecast is from the variation of the actual series.
• The covariance proportion measures the remaining unsystematic forecasting errors.
Note that the bias, variance, and covariance proportions add up to one.
If your forecast is "good", the bias and variance proportions should be small so that most of the bias should be concentrated on the covariance proportions.

3.0 Empirical Analysis

Data Properties:

Figure 1 indicates that the series contains trend components. To remove the trend components, we take the first difference (d) of the logarithms (I) of the data and the series are preferred in analysis of financial time series because they have attractive statistical property which is stationarity as shown in figure 2.
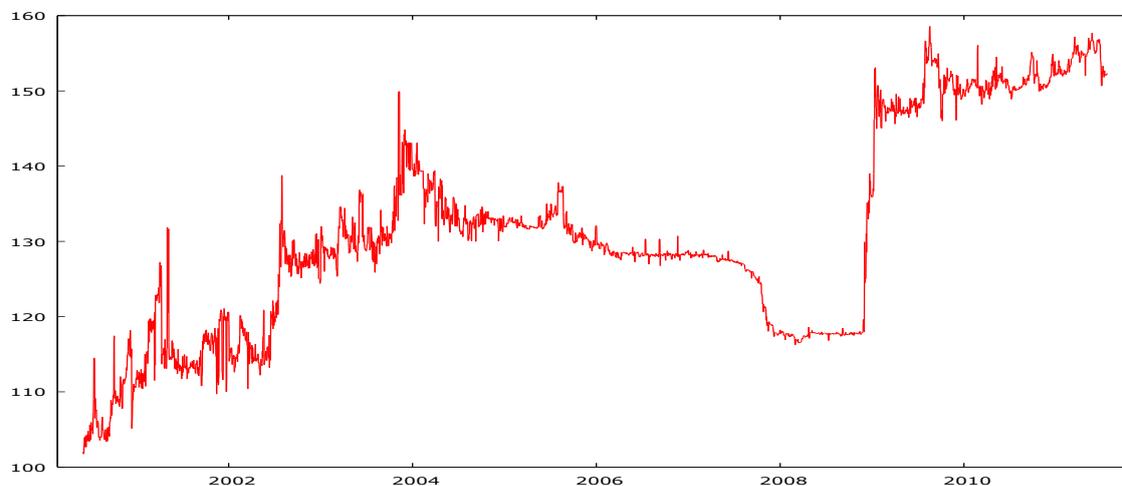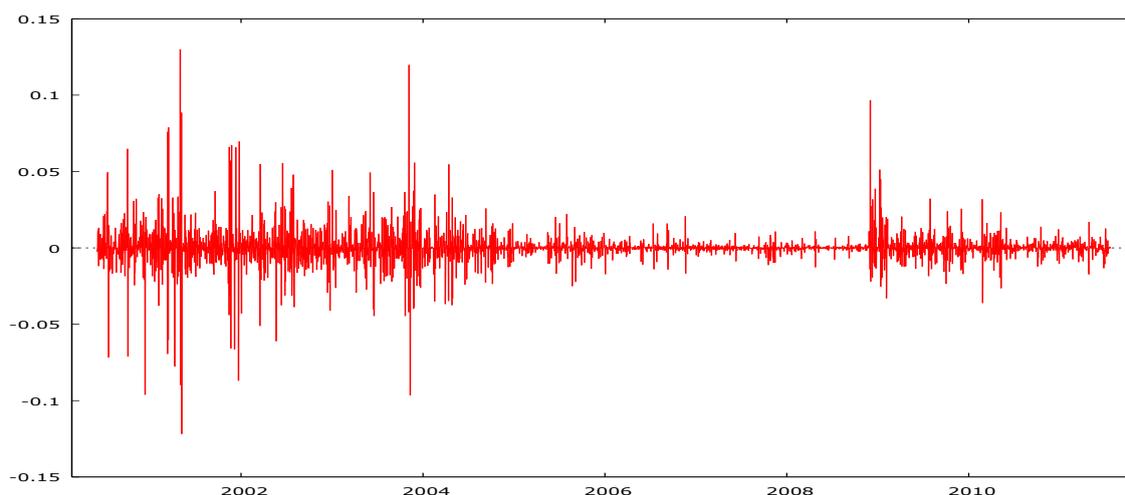

Figure 1

Figure 2

3.1 UNIT ROOT TEST FOR THE EXCHANGE RATE

The ADF statistic test the null hypothesis of unit root against the alternative of no unit root and the decision rule is to reject the null hypothesis when the value of the test statistic is less than the critical value. The KPSS statistic tests the null hypothesis of stationarity against the alternative of non stationarity and the decision rule is to accept the null hypothesis when the value of the test s tatistic is less than the critical value. The results of the ADF and KPSS tests are in Table 4.1 below.

Table 3.1 Results of the Unit Root test for the Exchange Rate.

Critical Values	ADF Test Statistics: -45.6949	KPSS Test Statistics: 0.0284
1%	-3.96	0.216

5%	-3.41	0.146
10%	-2.57	0.119

Table 3.1 The ADF test statistic is greater than all the critical values in absolute value so the hypothesis of non-stationarity is rejected. And for KPSS test statistic is less than the critical value, so the hypothesis is accept.

3.2 JARQUE BERA TEST FOR NORMALITY.

To achieve the first objective of the research, we examine the characteristics of the unconditional distribution of the exchange rate. This will enable us to explore and explain some stylized facts embedded in the financial time series. Jarque Bera normality test is used to demonstrate this and the results are given in Table 3.3 below:

Table 3.3 Jarque – Bera Test for Normality

Std.dev.	0.009954
Skewness	0.3795
Kurtosis	42.9911
Jakue Bera Test	271377
P- value	0.0000

Table 3.3 the results indicate the positive mean of daily exchange rate between USD/NGN, and standard deviation appear to be higher which follow the introduction of market determine exchange rate. The skewness is positively skews relative to the normal distribution (0 for the normal distribution). This is an indication of a non symmetric series. The kurtosis is very much larger than 3, the kurtosis for a normal distribution. Skewness indicates non-normality, while relatively large kurtosis suggests that distribution of the exchange rate return series is leptokurtic (i.e exhibit fat tail), Jarkue-Bera normality test statistics, indicating that neither return series has normal distribution.

3.3 ANALYSIS OF SOME VOLATILITY MODELS

Table 3.4: Parameter Estimates of GARCH models for the Period, June 2000 – July 2011

	GARCH(1,1)	GJR-GARCH(1,1)	TGARCH(1,1)	TS-GARCH(1,1)	EGARCH(1,1)	APARCH(1,1)

α_o	1.483e-07	3.1138e-07 (1.8694e-06)	2.6358e-07	1.8672e-07 (3.7609e-07)	-0.49869 (0.0000)	1.89e-06 (0.0000)
α_1	((3.2433e-07)	1.1485 (8.7710)	(3.1639e-07)	0.2725 (0.1919)	0.398305 (0.0000)	0.251253 (0.0000)
γ_1	0.5576 (1.2791)	-0.3312 (6.5431)	0.2427 (0.1568)		-0.013364 (0.0000)	-0.006686 (0.9836)
			-0.7970 (0.3044)			
β_1		0.9732 (0.0159)		0.9647 (0.0149)	0.970879 (0.0000)	0.836538 (0.0000)
Δ			0.9681 (0.1187)			1.773792 (0.0000)
Persistence	0.9755 (0.0114)				0.0771	0.0775
		1.9561		1.2372		
	1.5331		0.81238			
AIC	-33337.26213	-33347.01232	-33392.98844	-33370.04158	-33336.34907	-33336.34907
SI C	-33305.70391	-33309.14245	-33355.11858	-33338.48336	-33326.33937	-33327.3427
LL	16673.63106	16679.50616	16702.49422	16690.02079	16634.35221	16653.43231

From table 4.9 show that α_1 coefficient is not statistically significant in the GARCH and GJR-GARCH models but significant at the 5% level in TGARCH, EGARCH, APARCH and TS-GARCH models. This appears to show the presence of volatility clustering in TGARCH, APARCH, EGARCH and TS-GARCH models. Conditional volatility for these models tends to rise (fall) when the absolute value of the standardized residuals is large (smaller).

The coefficient of β$_1$ (a determinant of the degree of persistence) are statistically significant in all the models. The sum of α_1 and β_1 in the GARCH model exceed 1. this appears to show that shocks to volatility are very high. The GJR-GARCH models that is $\alpha_1 + \beta_1 + (\gamma_1/2)$ exceed 1. This also appears to show that shocks to volatility are very high and the variances are not stationary under The GJR-GARCH model. But under EGARCH, APARCH and TGARCH, model $\alpha_1 + \beta_1 + (\gamma_1/2)$ is less than 1 showing persistent volatility in the EGARCH, APARCH and TGARCH model. The sum of α_1 and β_1 in the TS-GARCH model exceeds 1. This appears to show that shocks to volatility are very high and will remain forever as the variances are not stationary under TS-GARCH model. So however, in sum, the Nigeria exchange rate market is characterized by high volatility persistence.

The γ_1 coefficient is asymmetry and leverage effects, are negative and statistically significant at the 5% level in all models. However, leverage effect will only exist if γ_1 >0 .Therefore, the hypothesis of leverage effect is rejected for all models but   asymmetry effect is accepted for the GJR-GARCH and TGARCH models. The results from the asymmetry models rejected the hypothesis of leverage effect. That is the GJR-GARCH and TGARCH models show the existence of statistically significant asymmetry effect.

Conclusively, the TS-GARCH and TGARCH models are found to be the best models. Because they have maximum likelihood, lower AIC and lower BIC.

4.   Forecast Evaluation:

Good volatility models have the ability to forecast (validation) and capture the commonly stylized facts. The ability to do so will further testify the validity of such models.

The forecasting ability is subsequently assessed using the symmetric lost functions which are the Mean Absolute Error (MAE), Root Mean Absolute Error (RMAE), Mean Absolute Percentage Error (MAPE) and Theil inequality Coefficient (Table 4.1). The results of the out-of-sample comparisons of accuracy of forecasts show that TS-GARCH model provide the most accurate forecasts for future Naira-dollar exchange rate volatility.

Table 4.1: Comparison of The Accuracy Of Volatility Forecasts

	GARCH	GJR-GARCH	TGARCH	TS-GARCH	EGARCH	APARCH
RMSE	0.013344	0.013639	0.013314	0.013343	0.015139	0.014843
MAE	0.006282	0.006465	0.006264	0.006232	0.007483	0.007402
MAPE	70.75423	70.84911	70.74543	70.74423	70.94548	71.17694
Theil-IC	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
Var Prop	0.999913	0.999888	0.999914	0.999913	0.999841	0.999728
Bias Prop	0.000087	0.000112	0.000086	0.000087	0.000159	0.000272
Cov Prop	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

5. Conclusion

The results show that the coefficient of ω (a determinant of the presence of volatility clustering) is statistically significant in the TGARCH and TS-GARCH models this appears to show the presence of volatility clustering. The forecasting ability is subsequently assessed using the symmetric lost functions which are the Mean Absolute Error (MAE), Root Mean Absolute Error (RMAE), Mean Absolute Percentage Error (MAPE) and Theil inequality Coefficient.

The results of the out-of-sample comparisons of accuracy of forecasts show that TS-GARCH model provide the most accurate forecasts for future Naira-dollar exchange rate volatility. This model will captured all the necessary stylize facts (common features) of financial data, such as persistent, volatility clustering and asymmetric effects. Ranking from the most accurate, we have TS-GARCH, TGARCH, GARCH, GJR-GARCH, APARCH and EGARCH respectively. Hence, we recommend the use of the models; these models may capture all the necessary stylize facts of financial data as the results suggested.

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